# Handbook Of Operational Amplifier Active RC Networks

Bruce Carter and L.P. Huelsman

ABSTRACT

While in the process of reviewing Texas Instruments applications notes, including those from the recently acquired Burr-Brown – I uncovered a couple of treasures, this handbook on active RC networks and one on op amp applications. These old publications, from 1966 and 1963, respectively, are some of the finest works on op amp theory that I have ever seen. Nevertheless, they contain some material that is hopelessly outdated. This includes everything from the state of the art of amplifier technology, to the parts referenced in the document – even to the symbol used for the op amp itself:
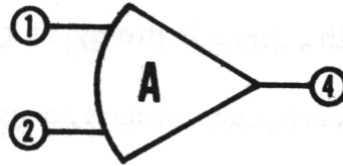
These numbers in the circles referred to pin numbers of old op amps, which were potted modules instead of integrated circuits. Many references to these numbers were made in the text, and these have been changed, of course.

In revising this document, I chose to take a minimal approach to the material out of respect for the original author - L.P. Huelsman, leaving as much of the original material in tact as possible while making the document relevant to present day designers. I did clean up grammatical and spelling mistakes in the original. I even elected to leave the original RC stick figure illustrations, which have minimal technical content – but added to the readability of the document.

Contents

Figures

Tables

ACTIVE RC NETWORK THEORY

The subject of active RC networks is one that has attracted considerable attention in the past few years from network theorists. Many new active devices and many new techniques have been developed. Some of these techniques have been of great theoretical interest, but of little practical value. Others, however, offer great practicality and have great potential for everyday application. In writing this hand- book, the goal has been to screen the large volume of literature on this subject, and present only those techniques that are of definite practical value to the working engineer.   All of the realization schemes described in Chapters 2 through 5 have been proven on the bench, and full details on their implementation are given in the "circuits" section of this handbook. In addition, each of these techniques is described in the text, where some of the pertinent theoretical background is given. The reader who is interested in a more detailed theoretical treatment will find that the references listed in Appendix A will give him an excellent introduction into the considerable literature on this subject.

CHAPTER 1

Introduction

This is a handbook on active RC networks. The first question about this subject that one might ask is, "What is an active RC network?" The answer is simple. It is collection of resistors, capacitors, and an active element (or elements). Viewed in another sense; it is a circuit without inductors. Why leave out inductors? There are many reasons. First of all, the inductor is a relatively large and heavy element. This is especially true at frequencies in the audio range and below.
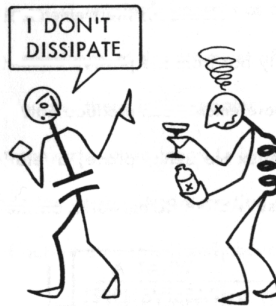
Second, inductors generally have more dissipation associated with them than capacitors of similar size. In other words, commercially available inductors are not nearly as "ideal" as commercially available capacitors. If you have tried to use network synthesis techniques you have probably discovered that the dissipation (or resistance) associated with inductors can cause considerable difficulty. For these reasons (and a few others such as non-linearity, saturation, and cost) more and more interest is being shown in circuit design techniques which avoid the use of inductors, namely active RC networks.

Can active RC networks do everything that passive RLC networks can do? Yes, and more! They can have natural frequencies any place in the left half at the complex frequency (or "s") plane. They can function as oscillators, in other words they can have natural frequencies on the jω axis. They can provide transformation ratios just like the coupled coils of a transformer do (however they can't provide the isolation). They can even provide perfect coupling and thus realize "ideal" transformers, which actual coupled coils cannot do. They can gyrate microfarads of capacitance into hundreds of henries of inductance, etc. There won't be space in this handbook to cover all of the things that active RC networks can do. Instead, we'll try to show you in detail how to use them to do some of your more common filtering tasks. If you are interested in more specialized applications, some references are given in Appendix A.
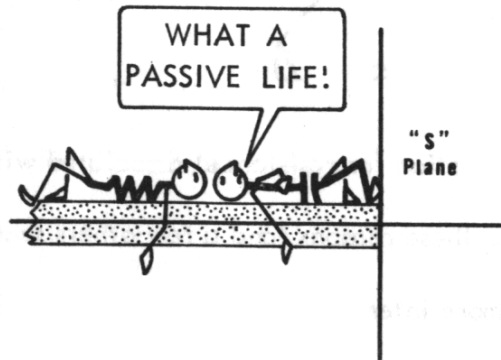
Natural Frequencies for Passive RC Circuits

How does the tremendous capability of active RC networks come about? Certainly not from the passive elements, the resistors and capacitors. Taken by themselves these elements can produce natural frequencies only on the negative real axis of the complex frequency plane, a relatively uninteresting region for most filtering applications. Active RC networks, on the other hand, can have natural frequencies anywhere on the complex frequency plane. Right half plane natural frequencies, of course, are not useful because they signify unstable network behavior, so we'll just consider the usable active RC natural frequencies as being in the left half plane or on the jω axis. Since it is the "active" element that gives active RC networks their potential, let's briefly consider such elements in more detail.
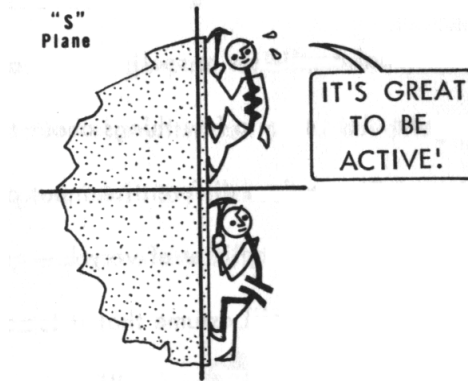
Stable Natural Frequencies for Active RC Circuits

There are several types of active elements that can be used in active RC networks. First, there is the ideal voltage amplifier of high gain. By "high" here we mean a gain in the order of at least 60db. By "ideal" we mean infinite input impedance and zero output impedance. The operational amplifier is an example of such an active element. Second, there is the ideal voltage amplifier of low gain. By "low" here we mean a gain in the order of 20 db or less. Such an element is sometimes referred to as a controlled source.

Third, there is the NIC (negative-immittance converter, also sometimes referred to as a negative-impedance converter). This is a two-port device (a device with two sets of terminal pairs) with the property that impedance connected across one set of terminals appears as negative impedance at the other set of terminals. Fourth, there is the gyrator, a device that converts capacitance to inductance and vice versa.
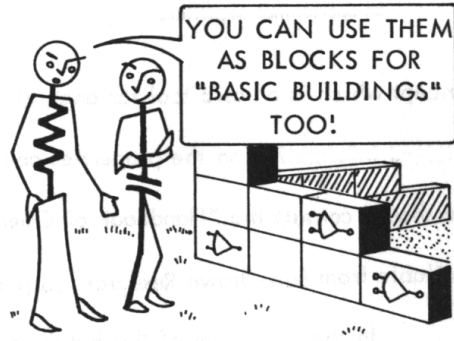
An interesting point to be noted here is that any of the last three types of active elements listed above can also be realized very simply and accurately with operational amplifiers. Thus, we see that the operational amplifier can be considered as a basic building black for constructing every type of active RC network. Many more details about the active elements introduced above will be given in the sections that follow. The networks that use operational amplifiers to realize these active elements will also be discussed. First, however, let us say a few things about the operational amplifier.

The modern differential input operational amplifier may be simply modeled as an ideal voltage amplifier of very low output impedance (we'll assume that it is zero), very high input impedance (we'll assume that it is infinite), and very high gain, with the property that the output voltage is proportional to the difference in the voltages applied to the two input terminals. An equivalent circuit for such a model is shown in Figure 1-1.
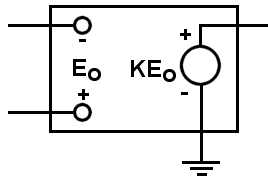
Figure 1-1. Model for an Ideal Operational Amplifier

The circuit symbol that will be used in future discussions is shown with the same terminal designation in Figure 1-2:
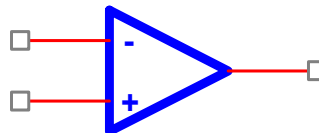
Figure 1-2. Circuit Symbol for an Operational Amplifier

As a result of the properties of the operational amplifier, when it is inserted in circuit configurations, the voltage between the input terminals - and + in Figures 1-1 and 1-2 is driven to zero. Due to the high input impedance and zero voltage, the current into both of these terminals may be considered as zero. These two characteristics comprise the "virtual ground" concept that is a basic tool for analyzing operational amplifier circuits. For more detailed information on the properties and characteristics of operational amplifiers, you should consult the "Handbook of Operational Amplifier Applications", SBOA092, which is available from Texas Instruments.

In the remainder of this handbook, we shall discuss in detail how the various types of active elements introduced above may be used to produce the most common types of network characteristics, namely, the low-pass, the high-pass, and the band-pass characteristics. We shall see that each of the active elements has advantages and disadvantages in the different circuit configurations. So, without more delay, let us start our investigation of active RC networks, a world without inductors.

CHAPTER 2

The Infinite-Gain Single-Feedback Circuit

The first active element that we shall consider for realizing active RC networks is the operational amplifier. In this chapter we shall investigate its use directly as an operational amplifier, in other words we shall not first modify it so that its characteristics approach those of some other active device. It may be helpful at this point to review briefly some of the characteristics of the operational amplifier. Those readers who are familiar with operational amplifiers may skip the next section without loss of continuity.

The Operational Amplifier
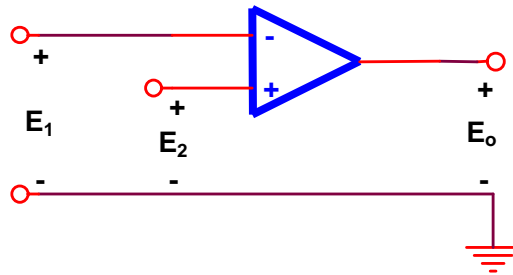
Figure 2-1. Symbolic Representation of the Operational Amplifier

In Figure 2-1 we have shown a symbolic representation for an operational amplifier that defines the input voltages E_1 and E_2 and the output voltage E_o. In terms of these voltages we may plot a typical open-loop DC transfer characteristics as shown in Figure 2-2.
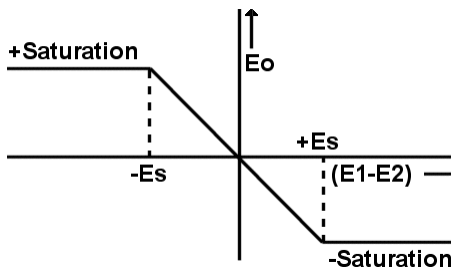
Figure 2-2. Open-Loop Transfer Characteristics of the Operational Amplifier

From this figure we see that the "-" terminal may be referred to as the "inverting" input terminal, while the "+" terminal may be referred to as the "non-inverting" input terminal. In a typical operational amplifier the magnitude of E_o is near the power supply rails at saturation. The open-loop DC gain of the amplifier shown in Figure 2-3 is 100000, so we see that the magnitude of Es, the differential input voltage that produces saturation, is only 100 μV. Therefore, open-loop operation of an operational amplifier is not practical. The open-loop frequency characteristic of a typical operational amplifier is shown in Figure 2-3.
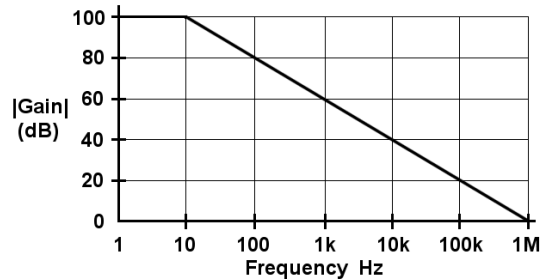
Figure 2-3. Open Loop Frequency Characteristic of a Typical Operational Amplifier

The slope of the roll-off is 20 dB/decade. A compensation network determines the location of the break point. In most voltage feedback operational amplifiers, this network is integral with the operational amplifier circuitry; in uncompensated amplifiers, the designer must supply it external to the amplifier packaging. Stability considerations determine the proper choice of compensation network for a given circuit configuration; however, most operational amplifiers are compensated so as to provide adequate performance for the majority of circuit applications. For additional information on stability, compensation, or other general properties of operational amplifiers, the reader is referred to the "Handbook of Operational Amplifier Applications", SBOA092, published by Texas Instruments.

The Basic Single Feedback Circuit

The basic circuit that will be considered in this chapter consists of two passive networks, which we will refer to as network A and network B, and an operational amplifier. Network A is connected between the input to the circuit and the input terminal of the operational amplifier Network B is used as a feedback network from the output to the input of the operational amplifier. The circuit is shown in Figure 2-4.
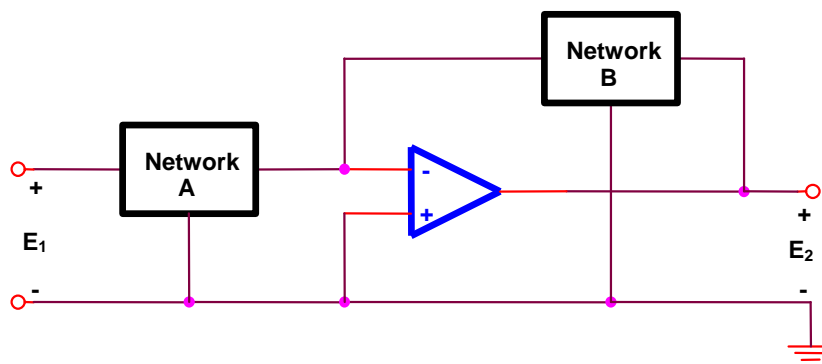
Figure 2-4. Basic Single Feedback Operational Amplifier Circuit

It should be noted that the operational amplifier is used in an inverting configuration, i.e., with its non-inverting input terminal (+) grounded. We shall call this circuit an *infinite-gain single-feedback circuit* since the operational amplifier that is the active element normally has very high gain, and since the feedback around it is made to a single point.

To characterize the properties of the two passive networks, we shall use their y parameters. For network A, we may define voltage and current variables as shown in Figure 2-5.
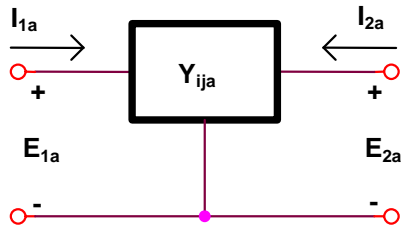
Figure 2-5. The Port Variables for Network A

The relations between these variables and the y parameters of the network are:

$$I_{1a} = y_{11a}E_{1a} + y_{12a}E_{2a}$$
$$I_{2a} = y_{12a}E_{1a} + y_{22a}E_{2a} \qquad (1)$$

Similarly, for network B and the variables shown in Figure 2-6:
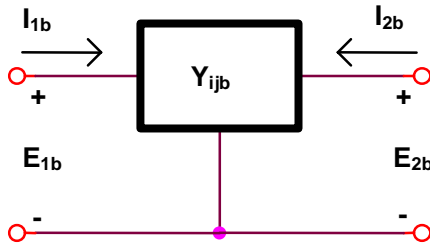
Figure 2-6. The Port Variables for Network B

$$I_{1b} = y_{11b}E_{1b} + y_{12b}E_{2b}$$
$$I_{2b} = y_{12b}E_{1b} + y_{22b}E_{2b} \qquad (2)$$

All of the voltage and current variables and the y parameters defined in equations (1) and (2) are functions of "s", the complex frequency variable.

The Voltage Transfer Function

The basic network configuration for the infinite-gain single-feedback circuit has been redrawn in Figure 2-7 to indicate the variables of the two passive networks explicitly.
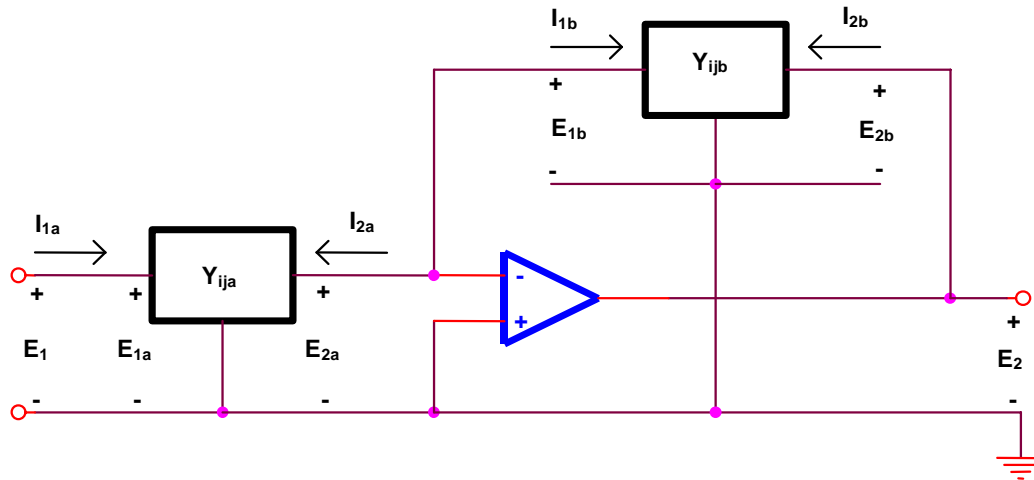
Figure 2-7. The Port Variables for the Basic Single-Feedback Circuit

In Chapter 1, it was pointed out that due to the "virtual ground," the voltage between the inverting and non-inverting terminals of the operational amplifier may be considered to be zero. Thus, the voltage E_{2a} shown in Figure 2-7 is zero. From the second equation of (1), we see that under this condition $I_{2a} = y_{12a}E_{1a}$. In addition, since E_{1a} and E_1 are equal, we may write:

$$I_{2a} = y_{12a}E_1 \qquad (3)$$

Similarly, for network B, E_{1b} is zero, and $E_{2b}=E_2$. Thus we see that:

$$I_{1b} = y_{12b}E_2 \qquad (4)$$

The virtual ground concept also tells us that the current into terminal 1 of the operational amplifier is negligibly small. Thus we see that $I_{2a} = -I_{1b}$. We may now combine equations (3) and (4) to obtain:

$$\frac{E_2}{E_1} = \frac{-y_{12a}}{y_{12b}} \qquad (5)$$

This is the open-circuit voltage transfer function for the infinite-gain single-feedback active circuit configuration.

Let us examine the voltage transfer function given in equation (5) more closely. If networks A and B are passive RC networks, their natural frequencies will be on the negative real axis of the complex frequency plane. Let us assume that both of the passive networks have the some natural frequencies; then the denominators of the functions y_{12a} and y_{12b} will cancel and the locations of these natural frequencies will not affect the voltage transfer function of the overall network. The poles of the voltage transfer function of the active network configuration will then be determined solely by the zeros of the transfer admittance y_{12b}. Since a passive RC network can have the zeros of its transfer admittance anywhere on the complex frequency plane, this says that we can realize complex conjugate poles in our voltage transfer function. Such poles will, of course, be restricted to the left half of the complex frequency plane for reasons of stability. Similarly, the zeros of the voltage transfer function given in equation (5) will be determined by the zeros of y_{12a}, and therefore we can realize any desired real or complex conjugate zeros in our voltage transfer function. Thus we see that an infinite-gain single-feedback active RC network configuration can be used to realize almost any desired pole-zero configuration.

One other property of this circuit should be noted. Suppose that another network with transfer admittance y_{12c} is also connected to the input terminal of the operational amplifier. The connection is shown in Figure 2-8, where the additional network is labeled as network C.
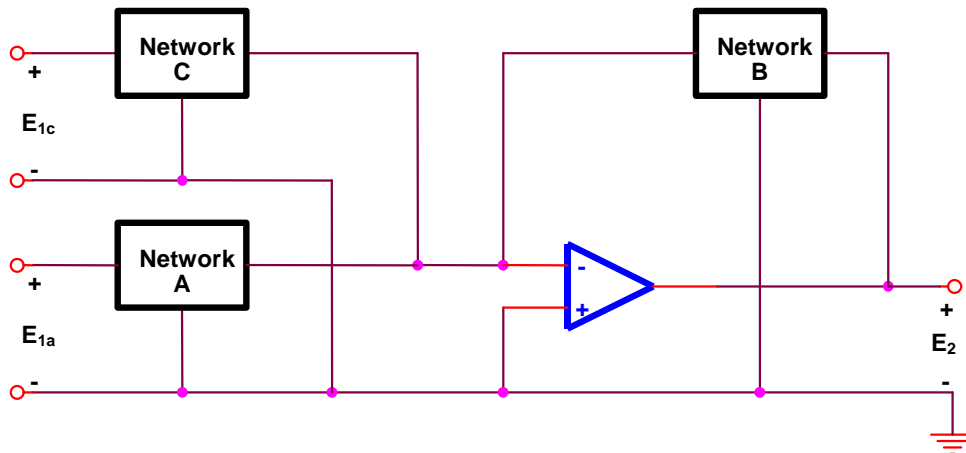
Figure 2-8. Dual Summing Single-Feedback Circuit

The input voltages to networks A and C are E_{1a} and E_{1c} respectively. An analysis similar to the one made in the preceding paragraph shows that the output voltage E_2 for this circuit is given by the relation:

$$E_2 = -\left[\frac{y_{12a}}{y_{12b}} E_{1a} + \frac{y_{12c}}{y_{12b}} E_{1c} \right] \qquad (6)$$

Thus we see that the infinite-gain single-feedback circuit configuration can also be used for summing signals from separate sources. This can be done without any interaction occurring between the sources.

The Passive Networks

In general, most filter designs require the use of complex conjugate poles. To produce these by the active RC technique described in this chapter, we thus require passive networks that have transfer admittances with complex conjugate zeros. There are several such network configurations, of which the two most common ones are the bridged-T network and the twin-T network. It is beyond the scope of this handbook to analyze such networks in detail. For completeness, however, we will present a simple design procedure for each type of network in this section.
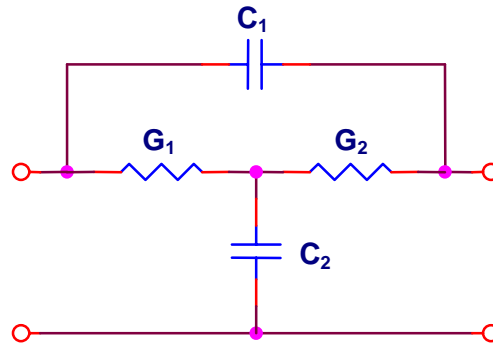
Figure 2-9. Bridged-T RC Network

An example of a bridged-T network is shown in Figure 2-9. The units for the elements of this network are farads for the capacitors and mhos (G=1/R) for the resistors. For a transfer admittance normalized to one radian/second, and of the form:

$$-y_{12} = \frac{s^2 + \alpha s + 1}{s + \alpha} \qquad (7)$$

The elements will have the following values:

$$C_1 = 0 \qquad\qquad (8)$$

$$G_1 = 2.5 - \alpha$$

$$G_2 = \frac{1}{\left(\alpha - \dfrac{1}{G_1}\right)}$$

$$C_2 = G_1 G_2$$

Such a network is not useful for producing zeros that lie close to the jω axis for small values of the constant α in the numerator of equation (7). A useful range of the constant a for this circuit is:

$$\tfrac{1}{2} < \alpha < 2 \qquad (9)$$

It should be noted that the usual frequency and impedance normalizations must be applied to this circuit to determine the actual element values.

An example of a twin-T network is shown in Figure 2-10.
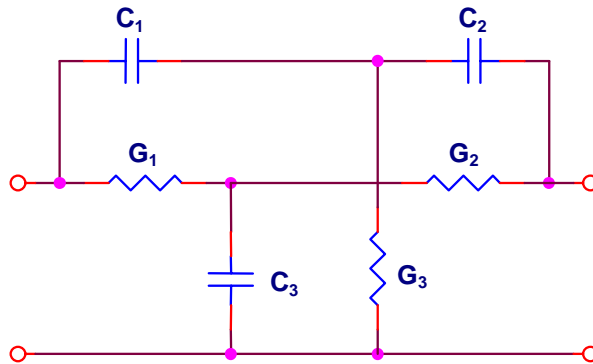
Figure 2-10. Twin-T RC Network

If the network elements are chosen such that:

$$A_1 = C_1 = G_1 = (5-\alpha)\frac{1+\alpha}{2+\alpha} \qquad (10)$$

$$A_2 = C_2 = G_2 = \frac{A_1}{(A_1 - 1)}$$

$$A_3 = C_3 = G_3 = \frac{A_1 A_2}{(1+\alpha)}$$

the transfer admittance of this network will be:

$$-y_{12} = \frac{(s+1)(s^2 + \alpha s + 1)}{(s+\sigma_1)(s+\sigma_2)} \qquad (11)$$

Note that in equations (10), the numerical values of the capacitors in farads and the resistors in mhos are equal. For convenience, these values have also been referred to as A_1, A_2, and A_3. The constants σ_1 and σ_2 in equation (11) can be found by the relations:

$$\sigma_1 = \frac{A_1 + A_2}{A_3} \qquad \sigma_2 = \frac{1}{\sigma_1} \qquad (12)$$

This network may be used for producing zeros that are as close to the jω, axis as desired, i.e., for as small a value of the constant α as may be required. Extremely small values of α, however, may lead to oscillations in the overall active circuit. It should be noted that the numerator of the transfer admittance of this network as given in equation (11) is of third degree with a negative real zero at –1. The poles of the transfer admittance, however, are located symmetrically about -1. This may be seen from the second equation of (12). It is easily shown that as the constant α approaches zero, σ_1 and σ_2 both approach unity. Thus, when α equals zero, cancellation occurs between one of the poles and the negative real zero in the numerator of the transfer admittance. Similarly, for small values of α, the numerator zero very nearly cancels one of the poles, and the transfer admittance may be assumed to have the form:

$$-y_{12} = \frac{s^2 + \alpha s + 1}{s+1} \qquad (13)$$

without significant loss of accuracy.

Network Design

Now let us consider the application of the infinite-gain single-feedback active RC circuit to the realization of three common filtering applications — the low pass, high pass, and band pass cases. For the low pass network, the frequency normalized voltage transfer function is of the form:

$$\frac{E_2}{E_1} = \frac{-H}{s^2 + \alpha s + 1} \qquad (14)$$

where H is a positive real constant giving the magnitude of the pass band gain. A common choice for the constant α is $\sqrt{2}$. This gives a maximally flat (sometimes called a Butterworth) frequency response characteristic. A bridged-T network of the type shown in Figure 2-9 may be used to produce the complex conjugate poles. The element values can be found from equations (8). The bridged-T network is used as network B of Figure 2-4. The transfer admittance for network A must have no finite zeros, but must have a single negative real pole. A network satisfying these requirements is shown in Figure 2-11.
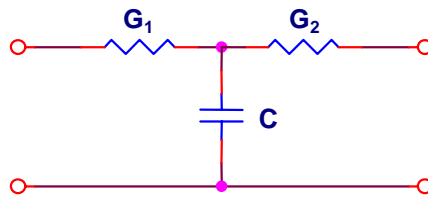
Figure 2-11. Low Pass Network A

This network has the transfer admittance:

$$-y_{12} = \frac{G_1 G_2 / C}{s + (G_1 + G_2)/C} \qquad (15)$$

Since this network must have the same pole location as network B, the factor $(G_1 + G_2)/C$ must be set equal to the constant α. The complete network, together with summary of the design procedure, and some sample element values is given in the circuit section of this handbook as Circuit No. 1.

For the high pass network, the frequency normalized voltage transfer function will be of the form

$$\frac{E_2}{E_1} = \frac{-Hs^2}{s^2 + \alpha s + 1} \qquad (16)$$

where H is a positive real constant giving the magnitude of the gain in the pass band. The same bridged-T network which was used for the low pass case may be used for network B for this case, since the poles of the transfer function will normally have similar locations The transfer admittance for network A must now have two zeros at the origin and a single pole on the negative real axis. A network satisfying these requirements is shown in Figure 2-12.
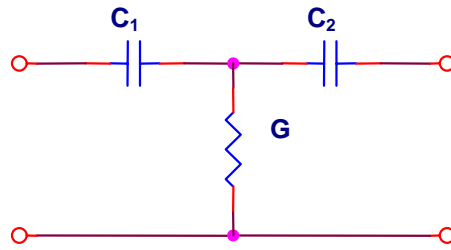
Figure 2-12. High Pass Network A

It has the transfer admittance

$$-y_{12} = \frac{s^2 C_1 C_2 / (C_1 + C_2)}{s + G/(C_1 + C_2)} \qquad (17)$$

The element values must be chosen so that the factor G/(C1 + C2) equals the constant α. The complete network, a design procedure, and a set of typical element values are given in the circuit section as Circuit No. 2.

For a band pass network, the frequency normalized voltage transfer function will be of the form

$$\frac{E_2}{E_1} = \frac{-Hs}{s^2 + \alpha s + 1} \qquad (18)$$

The magnitude of the gain in the pass band for this function is H/α, where H is a positive real constant. Most band pass filter applications require that the constant α be small, i.e., that the poles be close to the jω axis. For this case the bridged-T network is not satisfactory, and it will usually be necessary to use a twin-T network of the type shown in Figure 2-10 as network B. If we assume that the transfer admittance of the twin-T network is of the form given in equation (13), then the transfer admittance of network A must have a single zero at the origin and a pole at -1. A network satisfying these requirements is shown in Figure 2-13.

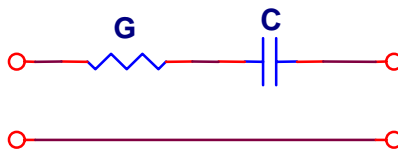**Figure 2-13. Single Zero – Single Pole Network A**

It has the transfer admittance:

$$-y_{12} = \frac{sG}{s + G/C} \qquad (19)$$

The factor G/C must be set equal to unity. The complete network, a design procedure, and a set of typical element values are given in the circuit section of this handbook as Circuit No. 3.

It should be noted that, in any realization produced by the method outlined in this chapter, the gain constant of the complete network realization is easily adjusted by changing the impedance normalization of either of the component passive networks, This is easily seen from equation (5). For example, to raise the overall network gain, one may either lower the impedance level of network A (and thus raise the magnitude of y_{12a}) or raise the impedance level of network B (and thus lower the magnitude of y_{12b}). It should also be noted that all circuits described in this chapter produce a signal inversion in addition to the frequency dependent properties that have been noted.

Conclusions

At this point we may make same conclusions regarding some of the properties of the infinite-gain single-feedback active RC circuit configuration described in this chapter. These conclusions will assist us in determining the relative merits of this configuration as compared to the other configurations that will be described in the chapters that follow. One of the major disadvantages of this active RC circuit configuration is the large number of passive elements that it requires. For example, in the band pass network described as Circuit No. 3, we see that eight elements are needed. Another difficulty is brought about by the fact that bridged-T and twin-T networks must be used to produce the complex conjugate poles. This means that any adjustment or trimming of the pole locations will be difficult since the passive elements interact to a high degree in such networks. On the positive side is the fact that this configuration has its pole locations determined completely by the passive networks. Thus the pole locations will remain relatively stable and independent of changes in the active element. This is a considerable advantage when it is desired to design high-Q networks, where the poles are located close to the jω axis, since even small pole displacements may produce instability in this case. Another advantage of this configuration is that the output impedance of the network is equal to the output impedance of the operational amplifier, which with high loop gain is very low.

Thus, this circuit may be used to drive other networks, without the need for an isolating stage, and without appreciable change in the circuit characteristics due to loading. Yet another desirable feature is the capability of summing signals at the input.

CHAPTER 3

The Infinite-Gain Multiple-Feedback Circuit

In the preceding chapter an active RC network configuration described as an infinite-gain single-feedback circuit was presented. An operational amplifier was used as the active element, and a single feedback path was provided around it. This chapter another active RC network configuration will be presented. An operational amplifier will again be used as the active element; however, more than one feedback path will be provided around it. The advantages and disadvantages of the two approaches will be compared.

The Basic Multiple Feedback Circuit

The basic circuit that will be described in this chapter consists of a number of two-terminal passive elements, interconnected so as to form feedback paths around an operational amplifier. The operational amplifier is used in an inverting configuration, with its non-inverting input terminal grounded. The general circuit configuration is shown in Figure 3-1.

Legend: ▭──▭ = Two Terminal Passive Element
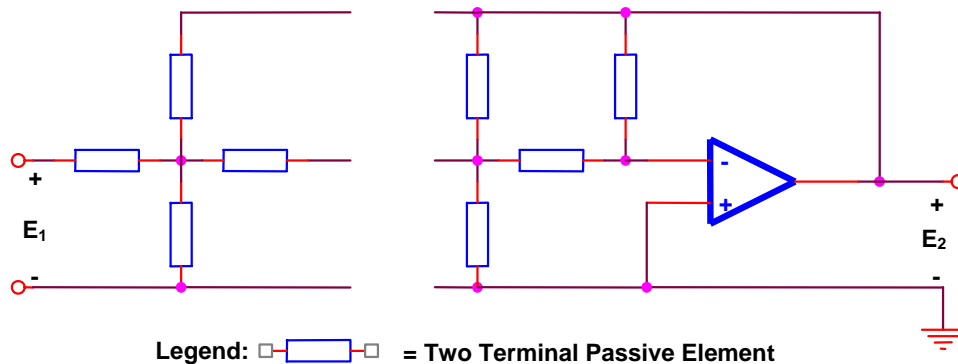
Figure 3-1. Multiple-Feedback (MFB) Operational Amplifier Circuit

We shall call this circuit an infinite-gain multiple-feedback circuit. In applying this circuit to the realization of transfer functions, it is practical to restrict each of the passive two-terminal elements to a single resistor or a single capacitor. In addition, if we limit ourselves to the realization of a voltage transfer function with a single pair of complex conjugate poles, and with zeros located only at the origin of the complex frequency plane or at infinity, then a maximum of five elements is necessary. The low pass, high pass, and band pass cases are included in this class of transfer functions. The extension of the method to other cases will be clear from the discussion that follows.

The Voltage Transfer Function

The basic circuit that may be used to realize voltage transfer functions with a single pair of complex conjugate poles and with zeros restricted to the origin or infinity is shown in Figure 3-2.

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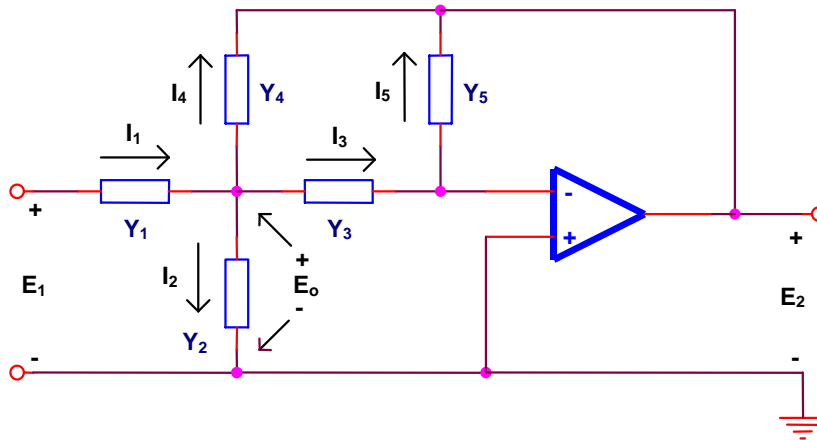
Figure 3-2. Basic Multiple-Feedback Circuit

Each of the elements Y_j shown in this figure represents a single resistor or a single capacitor. Reference currents I_j and an interior voltage E_o have been defined in the figure to aid in the analysis. From the figure we may write:

$$E_1 = \frac{1}{Y_1} I_1 + E_o \qquad (1)$$

It is also obvious from Kirchoff's law that:

$$I_1 = I_2 + I_3 + I_4 \qquad (2)$$

The virtual ground imposed by the operational amplifier requires that the voltage across both Y_2 and Y_3 equal E_o. Similarly, the voltage across Y_4 is the difference between E_o and E_2. Thus we may write expressions for the branch currents as:

$$I_2 = Y_2 E_o \qquad (3)$$
$$I_3 = Y_3 E_o$$
$$I_4 = Y_4 (E_o - E_2)$$

The virtual ground also requires that $I_3 = I_5$, thus we may write:

$$I_3 = Y_3 E_o = -Y_5 E_2 = I_5 \qquad (4)$$

If we substitute the relations of equations (2), (3), and (4) into equation (1), we obtain:

$$\frac{E_2}{E_1} = \frac{-Y_1 Y_3}{Y_5(Y_1 + Y_2 + Y_3 + Y_4) + Y_3 Y_4} \qquad (5)$$

This is the open-circuit voltage transfer function for the infinite-gain multiple-feedback circuit shown in Figure 3-2. The elements Y_j of this network may readily be chosen so as to obtain low pass, high pass, and band pass voltage transfer functions. This will be shown in the next section.

Network Design

Let us first consider the use of the infinite-gain multiple-feedback configuration to realize a low pass network. It is desired to obtain a frequency normalized voltage transfer function of the form:

$$\frac{E_2}{E_1} = \frac{-H}{s^2 + \alpha s + 1} \qquad (6)$$

where H is a positive real constant which specifies the gain in the passband, i.e., the dc gain. If we compare the above equation with equation (5), we see that in order to have the numerator not be a function of "s", both of the elements Y_1 and Y_3 must be resistors. Similarly, in order to generate the s_2 term in the denominator, Y_5 must be a capacitor, as must either Y_2 or Y_4. Y_4, however, must be a resistor; otherwise, it will not be possible to realize the constant term in the denominator (this must come from the product Y_3Y_4). Thus, we must make the following choices for the elements of the circuit shown in Figure 3-2:

$$Y_1 = G_1 \qquad (7)$$
$$Y_2 = sC_2$$
$$Y_3 = G_3$$
$$Y_4 = G_4$$
$$Y_5 = sC_5$$

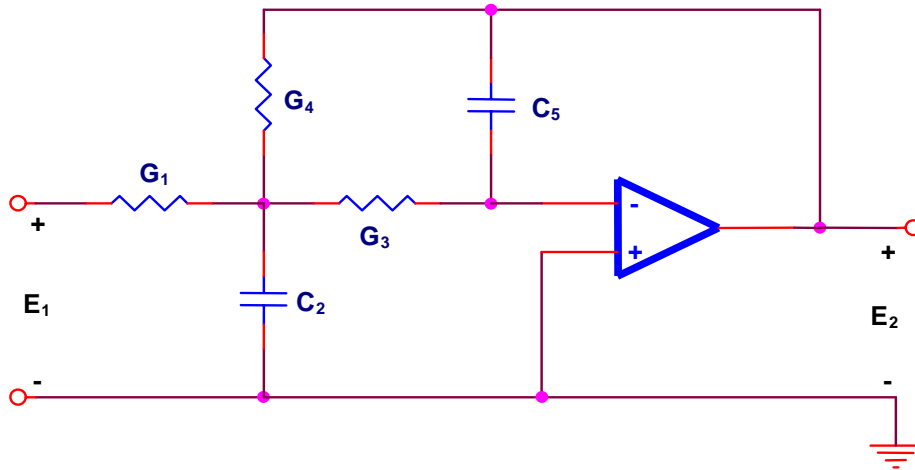The circuit with these elements is shown in Figure 3-3.

Figure 3-3. Low Pass MFB Filter

The voltage transfer function for this circuit is:

$$\frac{E_2}{E_1} = \frac{-G_1G_3}{s^2C_2C_5 + sC_5(G_1 + G_3 + G_4) + G_3G_4} \qquad (8)$$

It should be noted that this circuit produces a signal inversion, as will be true for all the circuits realized by this technique. The specific solutions for the element values in terms of the constants α and H may be found by equating corresponding coefficients in equations (6) and (8). Such a process leads to a simultaneous set of equations that are, unfortunately, non-linear. The nature of the set of equations is such, however, that constraints may be applied to develop a set of solutions. Such a set of solutions, together with other design information, is given in the circuit section of this handbook as Circuit No. 4. It should be noted that although the solutions given have been found to give good experimental results, they are not unique; i.e., other sets of solutions also exist.

The high pass network can be considered in a manner similar to the low pass network. The frequency normalized high pass voltage transfer function will be of the form:

$$\frac{E_2}{E_1} = \frac{-Hs^2}{s^2 + \alpha s + 1} \qquad (9)$$

where H is a positive real constant which specifies the gain in the pass band, i.e. the high frequency gain. The network elements shown in Figure 3-2 must be chosen as follows:

$$Y_1 = sC_1 \qquad (10)$$
$$Y_2 = G_2$$
$$Y_3 = sC_3$$
$$Y_4 = sC_4$$
$$Y_5 = G_5$$

The resulting voltage transfer function may be expressed in terms of the network elements as:

$$\frac{E_2}{E_1} = \frac{-s^2 C_1 C_3}{s^2 C_3 C_4 + sG_5\left(C_1 + C_3 + C_4\right) + G_2 G_5} \qquad (11)$$

The circuit configuration is shown in Figure 3-4.

Figure 3-4. High Pass MFB Active Filter

The same comments that were made with respect to solving for the element values in the low pass network also apply in this case. A set of design equations and a summary of other information on this circuit are given in the circuits section of this handbook as Circuit No.5.

There are several configurations of five elements that can be used to realize a band pass network with a frequency normalized transfer function of the form:

$$\frac{E_2}{E_1} = \frac{-Hs}{s^2 + \alpha s + 1} \qquad (12)$$

where H is a positive real constant and H/α is the magnitude of gain in the pass band. One of the most practical configurations is the one defined by the following choice of elements:

$$Y_1 = G_1 \qquad (13)$$
$$Y_2 = G_2$$
$$Y_3 = sC_3$$
$$Y_4 = sC_4$$
$$Y_5 = G_5$$

For such a choice, the voltage transfer function may be written in terms of the elements as

$$\frac{E_2}{E_1} = \frac{-sG_1C_3}{s^2C_3C_4 + sG_5(C_3 + C_4) + G_5(G_1 + G_2)} \qquad (14)$$

The circuit configuration is shown in Figure 3-5.

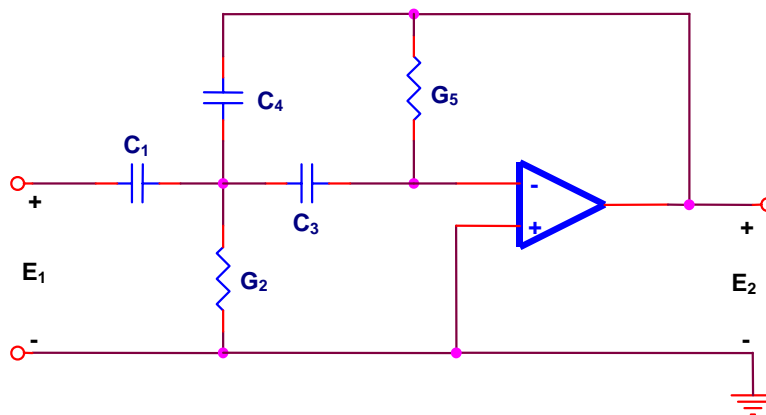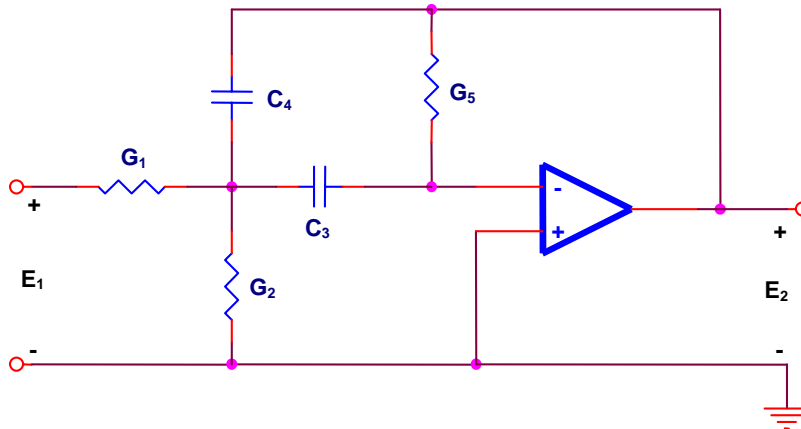**Figure 3-5. Band Pass MFB Active Filter**

A set of non-linear equations must be solved for the element values. The solutions and the circuit design information are given in the circuits section of this handbook as Circuit No. 6.

Conclusions

In a manner similar to that of the last chapter, we may make some conclusions regarding the characteristics of the infinite-gain multiple-feedback circuit configuration. One advantage we see is that the number of elements is greatly reduced from the number required for the infinite-gain single-feedback circuits of the preceding chapter. For example, for the low pass and high pass circuits, a maximum of five elements is required, compared with seven for the single-feedback circuit, As may be seen from the design equations given in the circuits section of the handbook, there are also cases in which one of the five elements may be eliminated in the band pass case, four or five elements are required rather than eight, a considerable savings. The same advantage of low output impedance which was pointed out for the single-feedback circuit holds true for the multiple-feedback circuit, since the output impedance of the circuit is just the closed loop output impedance of the operational amplifier. Thus this circuit may be used to drive other circuits without degradation of performance due to loading effects.

The multiple-feedback circuit, however, has same disadvantages that the single-feedback circuit does not. For example, it is not possible to obtain high Q band pass realizations with the multiple-feedback configuration without resorting to large spreads of element values. Another disadvantage is the fact that since feedback is made to two points, there is no one single point in the circuit which can be used to sum separate signals as could be done in the single-feedback configuration. Also, if it is desired to realize transfer functions with zeros other than at the origin or infinity, the networks and the design procedure for the multiple-feedback case become considerably more complicated. Finally, this approach, in general, cannot be used to achieve as large a value of gain constants as may be obtained by the single-feedback configuration. Articles have appeared in the literature discussing the application of the infinite-gain multiple-feedback circuit configuration to the realization of more complicated transfer functions. For the reader who wishes to pursue this topic further, some references are given in Appendix A.

CHAPTER 4

The Controlled Source Circuit

In the preceding chapters, some general properties of active RC circuits were introduced, and two types of active RC circuit configurations were analyzed. Both of these configurations required an active element with a high value of gain. Thus, they were referred to as "infinite-gain" realizations, and an operational amplifier was used to provide the gain. In this chapter a quite different circuit configuration will be presented. It requires on active element with a relatively low value of gain, which we shall refer to as a controlled source.

In general, a controlled source is an active network element which has an output voltage or current which is a function of some single input voltage or current, but is unaffected by any of the other voltages or currents in the network. There are four types of controlled sources: the voltage-controlled voltage source, the current-controlled voltage source, the voltage-controlled current source, and the current-controlled current source. Certain physical devices have characteristics that make them act in a manner similar to some of these sources. For example, a transistor acts somewhat like a current-controlled current source. For those who remember the "fire bottle" (tube) days of electronics, a pentode acted very nearly like a voltage-controlled current source.

The Voltage-Controlled Voltage Source

In our discussion of the use of controlled sources in active RC circuits, we will limit ourselves to a single type of source, an ideal voltage amplifier with infinite input impedance, zero output impedance, and an output voltage which is equal to the input voltage multiplied by same positive or negative constant, This device we shall refer to as a VCVS (voltage-controlled voltage source). A circuit model for it is shown in Figure 4-1.
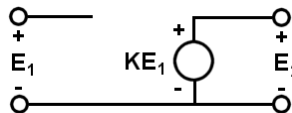
Figure 4-1. VCVS Circuit Model

From this figure we see that $E_2 = KE_1$, where the constant K is usually referred to as the "gain." It may be either positive or negative. The circuit symbol that we shall use for the VCVS is shown in Figure 4-2:

Figure 4-2. VCVS Circuit Symbol

Now let us see how we can realize a VCVS with an operational amplifier. For the values of K that are positive and greater than unity, the circuit shown in Figure 4-3 may be used.

Figure 4-3. Non-Inverting Operational - Amplifier VCVS

Note that in this circuit, the non-inverting terminal of the operational amplifier is used as the input terminal. Using the virtual ground concept introduced in Chapter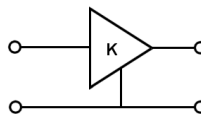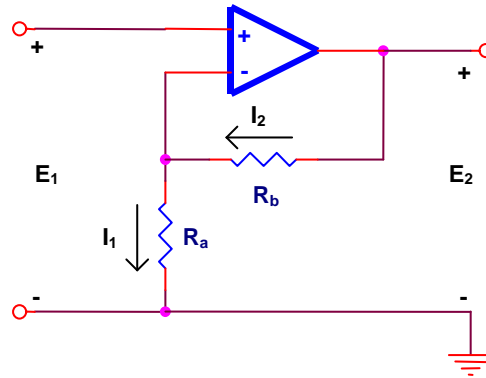 1, we see that the inverting and non-inverting inputs must be at the same potential. Since the voltage at the inverting input is equal to $R_a I_1$ the voltage at the non-inverting input must be the same; therefore, $E_1 = R_a I_1$. No current flows into the operational amplifier terminals; therefore, the currents I_1 and I_2 are equal. Thus $E_2 = (R_a + R_b)I_1$. Combining these relations we obtain:

$$\frac{E_2}{E_1} = \frac{R_a + R_b}{R_a} \qquad (1)$$

Since the input impedance at terminal 2 (as well as that at terminal 1) of the operational amplifier is very large, the circuit shown in Figure 4-3 may be considered as an ideal VCVS with a value of K equal to $(R_a + R_b)/R_a$. It should be noted that the idealness of this VCVS realization is relatively independent of the impedance levels that are chosen for R_a and R_b. For general-purpose applications, values of R_a in the range of 100 kΩ will usually prove satisfactory.

Network Design

The controlled source described in the last section may be used, in connection with passive RC networks, to obtain various network functions. The first such function that we shall consider is the transfer function for the low pass network. The frequency normalized voltage transfer function is:

$$\frac{E_2}{E_1} = \frac{H}{s^2 + \alpha s + 1} \qquad (2)$$

where H is a positive real constant giving the value of the gain in the pass band. A network configuration that will produce this function is shown in Figure 4-4.
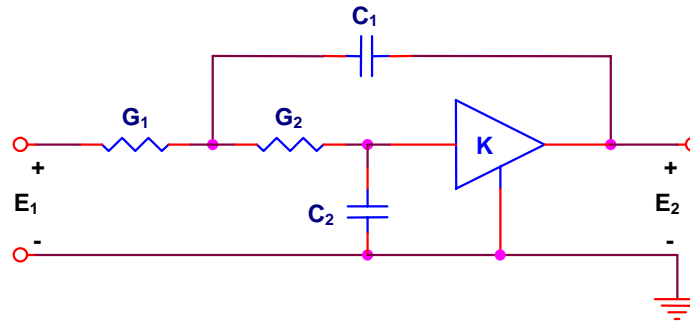
Figure 4-4. VCVS Low Pass Active Filter

In terms of the elements shown in this figure, we may write the voltage transfer function as:

$$\frac{E_2}{E_1} = \frac{KG_1G_2}{s^2C_1C_2 + s(C_2G_1 + C_2G_2 + C_1G_2 - KC_1G_2) + G_1G_2} \qquad (3)$$

From the preceding equations we see that no signal inversion is introduced by this network configuration.

The transfer function given in equation (3) points out some interesting properties of this circuit realization. First of all, the low frequency gain, the gain in the pass band, may be found by evaluating equation (3) in the limit as "s" approaches zero. It is readily seen that this gain is simply equal to K. In other words, the overall gain specified for the circuit directly determines the gain of the VCVS. Second, if it is desired to change the value of the cutoff frequency for this circuit, this may done by changing the values either of the resistors or the capacitors. Such changes will not affect the gain in the pass band. If, in addition, the changes are made in such away that the ratio of the two elements changed remains the same, i.e. if the same percentage change is made to each of them, then the relative shape of the magnitude and phase characteristics of the network will remain unchanged. Thus this filter has the property that its cutoff frequency can be readily shifted.

The values of the network elements must be found by simultaneously equating the coefficients of equations (2) and (3) The resulting set of equations is non-linear, but solutions for the element values in terms of the constants α and H are easily found set of solutions, together with other design information for the circuit is given in the circuits section of the handbook as Circuit No. 7.

The low pass circuit described above is sometimes modified to the configuration shown in Figure 4-5.
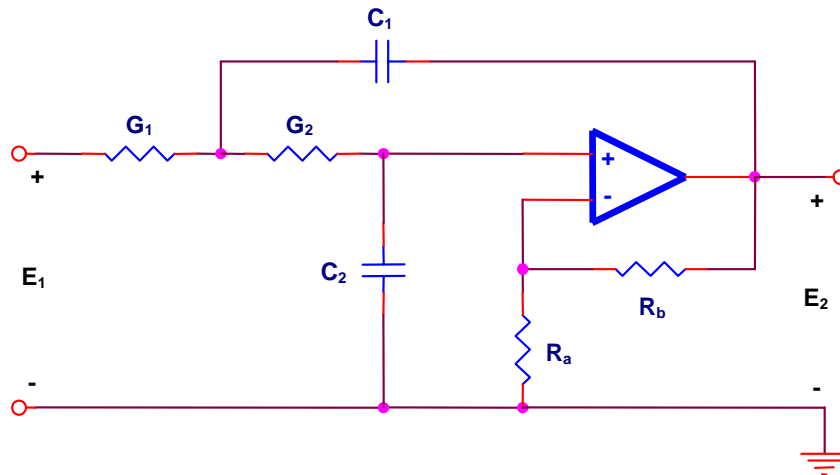
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Figure 4-5. Operational Amplifier VCVS Low Pass Active Filter

If this figure and Figure 4-4 are compared, it is easily seen that the right terminal of the capacitor C_1 has been moved from the output of the operational amplifier (which is also the output of the controlled source) to the junction of the two resistors R_a and R_b whose values determine the gain of the controlled source. If the magnitude of the impedance of the capacitor C1 is much larger than the values of the resistors R_a and R_b, so that the capacitor does not load the resistor network, then the voltage transfer function for the circuit shown in Figure 4-5 is:

$$\frac{E_2}{E_1} = \frac{KG_1G_2}{s^2C_1C_2 + s(C_2G_1 + C_2G_2) + G_1G_2} \qquad (4)$$

The above equation illustrates one advantage of this configuration; i.e. none of the coefficients in the denominator are a function of K, the gain of the controlled source. Thus K may be varied, changing the gain of the network, without changing the frequency characteristics of the network. Such a change of gain may, of course, be accomplished by varying either R_a or R_b in Figure 4-5. It should be noted that this circuit also has a disadvantage. Since there is no subtraction of terms in the coefficient of the first degree term in the denominator of equation (4), it is not possible to realize transfer functions of the type given in equation (2) in which the constant a has very small values.

The frequency normalized voltage transfer function for a high pass network is:

$$\frac{E_2}{E_1} = \frac{Hs^2}{s^2 + \alpha s + 1} \qquad (5)$$

A realization for such a transfer function using a VCVS as the active element is shown in Figure 4-6.
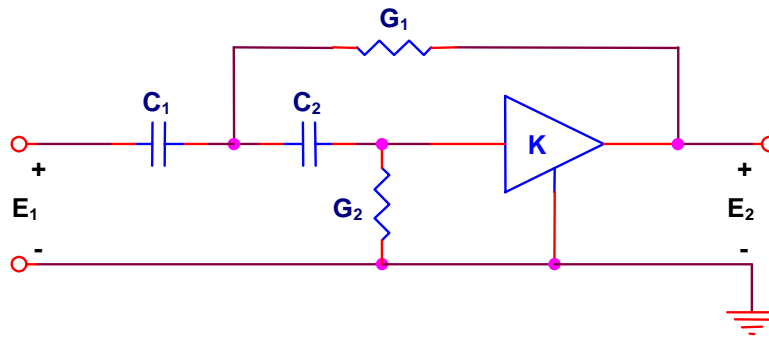
Figure 4-6. VCVS High Pass Active Filter

The voltage transfer function is:

$$\frac{E_2}{E_1} = \frac{Ks^2C_1C_2}{s^2C_1C_2 + s(C_1G_2 + C_2G_2 + C_2G_1 - KC_2G_1) + G_1G_2} \qquad (6)$$

The same comments that were made with respect to the low pass network also apply here; namely, H is positive, and the pass band gain (in this case the high frequency gain) is equal to the gain K of the VCVS and is not a function of the passive elements of the network. Similarly, the cutoff frequency can be changed by changing the values of the resistors or the capacitors, and, if equal percentage changes of the elements are made, the relative shape of the frequency characteristics of the network will remain unchanged. A set of formulas for determining the values of the network elements, together with other design information, is given in the circuits section of the handbook as Circuit No. 8.

In a manner similar to that which was done for the low pass network, the high pass configuration shown in Figure 4-6 may be modified to the configuration shown in 4-7, by moving the right terminal of the resistor labeled G_1 from the output of the controlled source to the junction of the two resistors R_a and R_b.
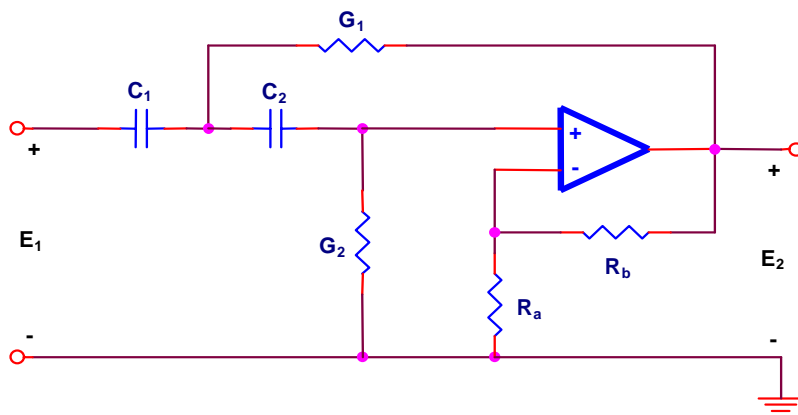

Figure 4-7. Operational Amplifier VCVS High Pass Active Filter

If the magnitude of the resistance of the element G_1 is much larger than the values of the resistors R_a and R_b, so that the gain-determining resistor network is not loaded, then the voltage transfer function far the circuit shown in Figure 4-7 is:

$$\frac{E_2}{E_1} = \frac{Ks^2C_1C_2}{s^2C_1C_2 + s(C_1G_2 + C_2G_2) + G_1G_2} \qquad (7)$$

Since none of the denominator coefficients in equation (7) are functions of K, the gain of the controlled source, this gain may be varied, changing the gain of the network without changing the frequency characteristics of the network. As in the law pass case is not possible to realize a voltage transfer function of the type given in equation (5) in which the constant α has very small values.

The frequency normalized voltage transfer function for a band pass network is:

$$\frac{E_2}{E_1} = \frac{Hs}{s^2 + \alpha s + 1} \qquad (8)$$

where H is a positive real constant and H/α is the gain in the pass band. There are several network configurations using a VCVS as the active element that may be used to realize such a transfer function. One such configuration that has been found to give good experimental results is shown in Figure 4-8:
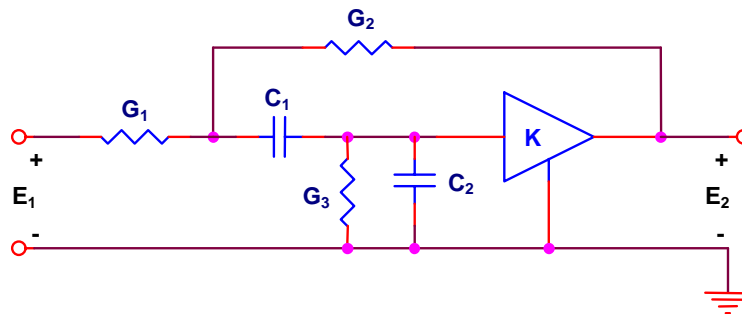

Figure 4-8. VCVS Band Pass Active Filter

The voltage transfer function in terms of the elements is:

$$\frac{E_2}{E_1} = \frac{sKC_1G_1}{s^2C_1C_2 + s(C_1G_3 + C_2G_1 + C_2G_2 + C_1G_1 + C_1G_2 - KC_1G_2) + G_3(G_1 + G_2)} \qquad (9)$$

It should be noted that in the denominator, K, the gain of the controlled source, appears only in the coefficient of the first-degree term. Thus, in high Q realizations, we may adjust the real part of the pole locations by varying K without appreciably affecting the resonant frequency. Since this adjustment may be made by varying the value of either of the resistors associated with the operational amplifier realization for the VCVS (see Figure 4-3), this is a very convenient means of adjusting the Q of the network. From equation (9) we see that such adjustments will also change the overall gain constant H, but this is usually a minor effect. The design information for this circuit is given in the circuits section of the handbook as Circuit No. 9.

Other Realizations with Voltage-Controlled Voltage Sources

Up to this point, in this chapter we have described active RC circuit configurations that have required a non-inverting VCVS. There are also many network configurations which require the use of an inverting VCVS, i.e., one in which K, the gain constant, is negative. An operational amplifier and the circuit shown in Figure 4-9 may produce such a source:
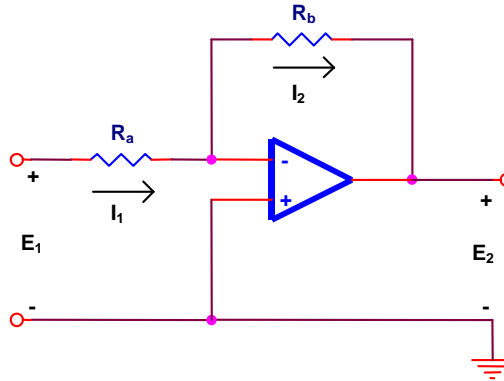

Figure 4-9. Inverting Operational Amplifier VCVS

Note that in this circuit the non-inverting input of the operational amplifier is grounded. If we apply the Q virtual ground concept to this circuit we see that $E_1 = I_1 R_a$. Similarly, $I_2 = I_1$ and therefore $E_2 = -I_1 R_b$. Thus we may write:

$$\frac{E_2}{E_1} = \frac{-R_b}{R_a} \qquad (10)$$

For large values of R_a, this circuit may be considered as an ideal VCVS with a value of K equal to $-R_b/R_a$. Since the resistor R_a must be chosen large, this places a limit an the maximum gain value that K may have, as well as on the frequency range aver which the circuit will effectively model the ideal VCVS. Actual values of R_a to be used will necessarily depend on the impedance level of the rest of the circuit, but values of the general order of 1 kΩ to 1 MΩ are not uncommon.

An example of the use of an inverting VCVS to realize a low pass transfer function as given in equation (2) is shown in Figure 4-10:
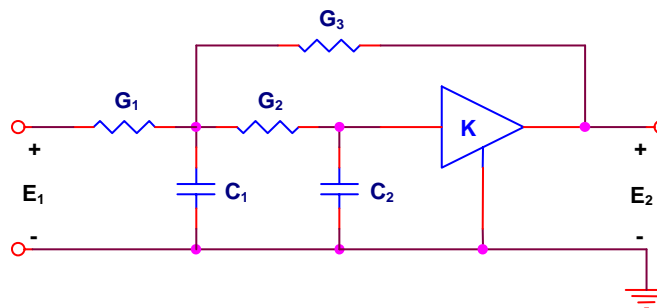

Figure 4-10. Inverting VCVS Low Pass Filter

The voltage transfer function for this network is:

$$\frac{E_2}{E_1} = \frac{-|K|G_1G_2}{s^2C_1C_2 + s[C_2(G_1 + G_2 + G_3) + C_1G_2] + G_2(G_1 + G_3 + |K|G_3)} \qquad (11)$$

To avoid misinterpretation, the negative sign associated with the constant K has been written into the equation, and the gain is expressed as a magnitude. In a similar manner the inverting VCVS may be used to realize a high pass network function of the type given in equation (5). The circuit is shown in Figure 4-11:
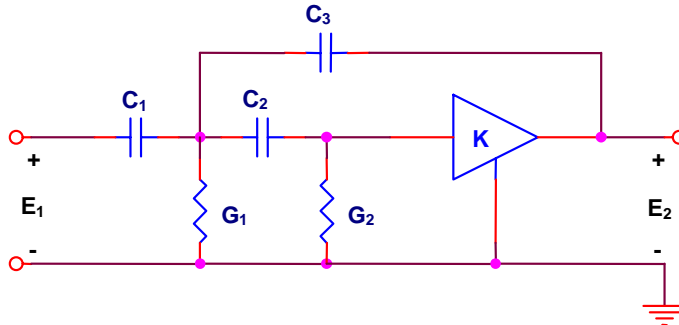

Figure 4-11. Inverting VCVS High Pass Filter

The voltage transfer function for this circuit is:

$$\frac{E_2}{E_1} = \frac{-|K|s^2C_1C_2}{s^2C_2(C_1 + C_3 + |K|C_3) + s[C_2G_1 + G_2(C_1 + C_2 + C_3)] + G_1G_2} \qquad (12)$$

The gain K has been shown as a magnitude in the same manner that was done in equation (11). It is also theoretically possible to use the inverting VCVS to realize a band pass transfer function. However, the value of the gain that is required for the source far even a moderately high value of Q is usually excessively high. Thus, such an application is of only limited value.

Conclusions

The active RC circuits using controlled sources which hove been described in this chapter have several advantages and disadvantages when compared to the circuits of the preceding chapters. First, a new and different variable appears in the transfer function equations, namely K, the gain of the controlled source. In the infinite-gain realizations given in Chapters 2 and 3, the network functions were almost completely unaffected by changes in the gain. In the realizations of this chapter, however, not only is there a strong dependence on the gain of the source, but also this effect may actually be used to vary the properties of the network. Thus, the presence of the gain as a variable is both an advantage and a disadvantage, and the potential user of these circuits must be aware of both.

The high and low pass networks that use the controlled source present another new and useful characteristic; i.e., the pass band gain is independent of the element values. This has considerable application for the case where the cutoff frequencies are to be varied without changing the gain It should also be noted that the circuits presented in this chapter have the same low output impedance that the infinite-gain realizations had. Thus, they may be used to drive other circuits without using isolating amplifier stages between them and the circuits that follow.

CHAPTER 5

The NIC In Active RC Circuits

In the first chapter of this handbook, the concept of an active device called an NIC (negative-immittance converter) was introduced. It was pointed out that an NIC could be used in connection with resistors and capacitors to realize a wide range of network functions. In this chapter we shall explore some of the properties of this device and see in detail how it can be used in active RC circuits.

The NIC (Negative-Immittance Converter)

Basically, the NIC is a two-port device that has the property that the impedance seen at either of its ports is the negative of the impedance connected to the other port. This "negative" action can come about in either of two ways. As a first way, the NIC can invert the direction of current flow with respect to that which would normally occur in a passive network, without disturbing the relative polarity of the input and output voltages. For example, consider a two-port network with a load impedance Z as shown in Figure 5-1:
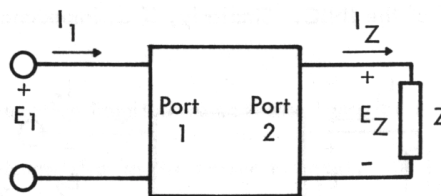

Figure 5-1. Two-Port Network With Load

If a current I_Z flows out of port 2 as shown, then we would expect that a current I_1 would flow into port 1 (assuming that the output voltage E_Z and the input voltage E_1 have the same polarity). If the two-port device somehow inverts one of these currents, then we have the situation wherein the Application of voltage E_1 to port 1 causes a current to flow in a direction opposite to that shown for I_1; i.e., it opposes the applied voltage. In other words, the input impedance is negative. More formally, such a device can be defined in terms of the two-part variables shown in Figure 5-2:
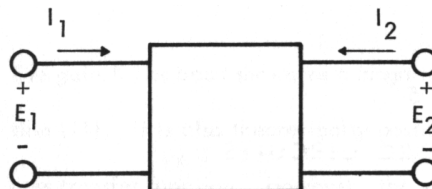

Figure 5-2. The Port Variables for a Two-Port Network

and the equations:

$$E_1 = E_2 \qquad (1)$$

$$I_1 = KI_2$$

These equations may be said to define an ideal current-inversion negative-immittance converter or INIC for short. The constant K is usually referred to as the "gain" of the INIC. In the next section we shall show that a single differential-input operational amplifier can easily realize such a device.

Let us investigate some properties of the INIC. First of all, consider the case where on impedance Z_2 is connected across the terminals of port 2 as shown in Figure 5-3.
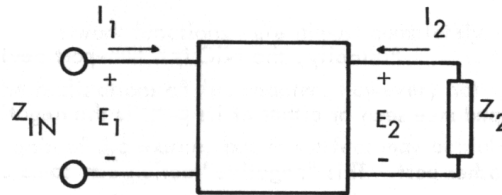

Figure 5-3. INIC Input Circuit

The variables of port 2 are then constrained by the relation $E_2 = Z_2 I_2$. Substituting this relation into equations (1) we see that:

$$Z_{IN} = \frac{E_1}{I_1} = -\frac{Z_2}{K} \qquad (2)$$

Thus the input impedance at port 1 is 1/K times the negative of the impedance connected across port 2. Thus we see that changing the gain K of the INIC easily varies the magnitude of this negative impedance. Similarly, if an impedance Z_1 is connected across port 1 of the INIC as shown in Figure 5-4:
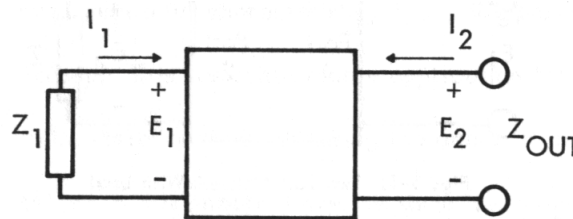

Figure 5-4. INIC Output Circuit

then the impedance ZOUT seen at port 2 may be shown to be:

$$Z_{OUT} = \frac{E_2}{I_2} = -KZ_1 \qquad (3)$$

It should be noted that again negative impedance is produced, but that in this case the gain constant K has the opposite effect to the one it had in equation (2). For the case where K = 1, the INIC will theoretically give the same results in either direction. Practically, however, stability considerations based on the network configuration in which the INIC is used usually do not permit interchanging ports 1 and 2 of the INIC, even for the unity gain case. More will be said about this when discussing the realization of the INIC.

A second way in which the "negative" action of an NIC can be brought about is by inverting the voltage while keeping the direction of current flow through the two-port device unchanged. In terms of the variables shown in Figure 5-2, this type of NIC is defined by the relations:

$$I_1 = -I_2 \qquad (4)$$

$$E_1 = -KE_2$$

Such a device may be referred to as an ideal voltage-inversion negative-immittance converter or VNIC for short. The constant K is called the "gain" of the VNIC.

Space does not permit developing some of the other properties of the NIC such as power relationships, impedance transformations, etc. The reader who is interested in learning more about this device should consult the references listed in Appendix A.

A Realization for the INIC

An INIC (ideal-current-inversion negative immittance converter) may be realized by using a differential-input operational amplifier as the active element. The circuit is shown in Figure 5-5.
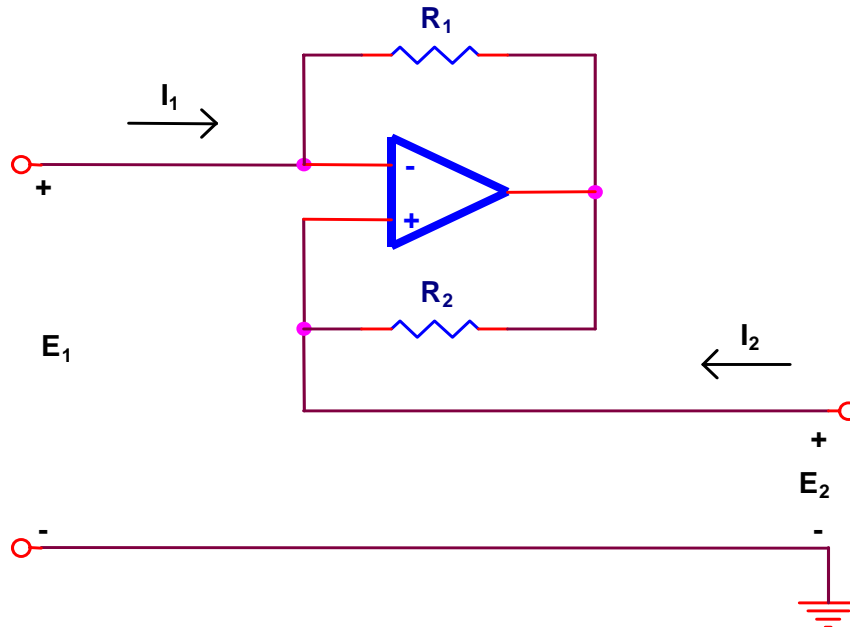

Figure 5-5. Operational Amplifier Realization of the INIC

We may analyze this circuit by means of the virtual ground concept introduced in Chapter 1. This concept tells us that the voltage between the inverting and non-inverting inputs of the operational amplifier is zero; thus we see that the voltages at the two ports of the overall network shown in Figure 5-5 are equal. Similarly, we know that no current flows into either of the amplifier inputs. Since the voltage across the two resistors and R_2 must be the same, we see that $I_1R_1 = I_2R_2$; in other words, the ratio of the currents at the two ports is determined by the ratio of the resistors. We may write the above relationships as:

$$E_1 = E_2 \qquad (5)$$

$$I_1 = \frac{R_2}{R_1}I_2$$

These are the same relations given in equations (1), with $K = R_2/R_1$. Thus we see that the circuit shown in Figure 5-5 has the properties of an INIC, and that the gain constant K may be easily adjusted by changing the values of either of the resistors R_1 or R_2.

Stability of the INIC

In the discussion given above, it was assumed that the voltage between the input terminals of the operational amplifier was zero. This assumption greatly simplified the analysis of the circuit. Actually, there will always be a small voltage present between these terminals. In the INIC circuit this small voltage becomes a significant factor in determining whether the circuit will be stable. To see this, consider the case where a resistor R_a is connected across port 1 of the INIC shown in Figure 5-5, and another resistor is connected across port 2. The circuit may be redrawn using the model for the operational amplifier shown in Figure 1-1 of Chapter 1. The result is shown in Figure 5-6.
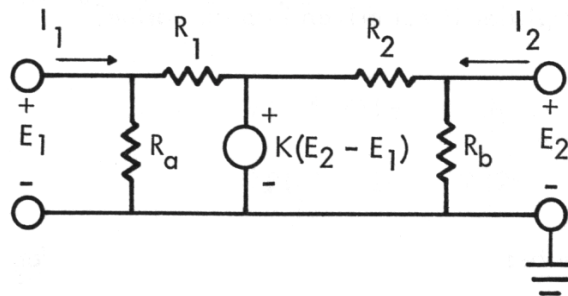

Figure 5-6. Circuit Model of the Operational Amplifier Realization INIC

The constant K shown in this figure represents the gain of the operational amplifier (not the gain of the INIC). In order for this circuit to be stable, the voltage $E_2 - E_1$, even though it is small, must never be positive. It if should go positive, the feedback provided by the resistor networks will drive the operational amplifier into saturation. Therefore, the condition for stability is:

$$E_1 \geq E_2 \qquad (6)$$

The voltages E_1 and E_2, however, if the currents I_1 and I_2 are zero, may be expressed in terms of the four resistors shown in Figure 5-6. Thus the inequality given in equation (6) may also be expressed as:

$$\frac{R_a}{R_1 + R_a} \geq \frac{R_b}{R_2 + R_b} \qquad (7)$$

This may be reduced to:

$$R_a R_2 \geq R_b R_1 \qquad (8)$$

For the case where R_1 and R2 are equal, corresponding with an INIC gain of unity, we see that:

$$R_a \geq R_b \qquad (9)$$

is a necessary condition for stable operation of the INIC. To indicate this, it is customary to refer to port 1 as the open-circuit-stable (OCS) port of this INIC realization, and port 2 as the short-circuit stable (SCS) part. Similar restrictions may be developed for the situation where the INIC is embedded in networks containing capacitors as well as resistors, to ensure that the operational amplifier is not driven into saturation.

The Basic INIC Circuit

There are several methods that have been proposed whereby a voltage transfer function may be realized by passive networks and an INIC. We shall discuss only one of these here, some others may be found in the references listed in Appendix A. The basic circuit configuration consists of two passive RC networks labeled A and B, and an INIC, and is shown in Figure 5-7.
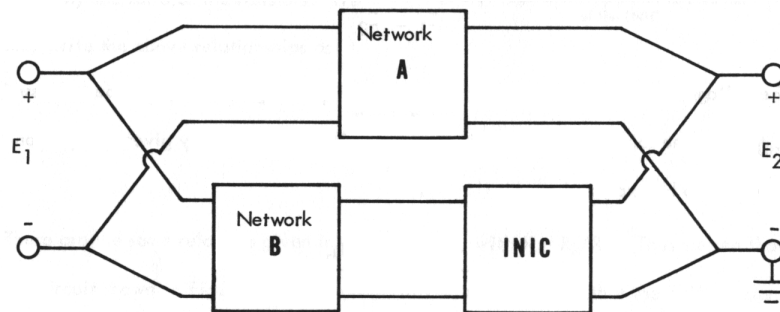

Figure 5-7. Basic Voltage Transfer Circuit Using the INIC

First let us consider the cascade connection of the B network and the INIC. If we describe the properties of the B network by its y parameters, then the effect of the INIC is to invert the current at the output port of network B and also multiply it by a constant. The y parameters of the cascaded connection of the B network and the INIC may thus be shown to be:

$$y_{11} = y_{11b}$$
$$y_{12} = y_{12b}$$
$$y_{21} = -Ky_{12b} \qquad (10)$$
$$y_{22} = -Ky_{22b}$$

where y_{11b}, y_{12b}, and y_{22b} are the y parameters of the B network by itself, and K is the gain of the INIC. Note that for the cascade connection, y_{12} is not equal to y_{21} as would be the case for a passive network. When the A network is connected in parallel, its y parameters add to those given in equations (10). Thus, the y parameters for the overall network are:

$$y_{11} = y_{11a} + y_{11b}$$
$$y_{12} = y_{12a} + y_{12b}$$
$$y_{21} = y_{12a} - Ky_{12b} \qquad (11)$$
$$y_{22} = y_{22a} - Ky_{22b}$$

Since the open-circuit voltage transfer function for an arbitrary network is simply the ratio -y_{21}/y_{22}, we may write the voltage transfer function of the basic circuit as:

$$\frac{E_2}{E_1} = \frac{-y_{12a} + Ky_{12b}}{y_{22a} - Ky_{22b}} \qquad (12)$$

In the references, some specific rules are developed for determining the parameters of the component passive networks A and B in such a way that they can be realized by passive RC networks for a given desired voltage transfer function. We shall not develop these rules here. In the next section, however, we shall show the results of applying such developments to obtain the some three types of transfer functions that we realized in earlier chapters.

Network Design

The basic circuit configuration described in the preceding section may be used to obtain various networks. The first network that we shall consider here is the low pass network having a frequency-normalized voltage transfer function of the form:

$$\frac{E_2}{E_1} = \frac{H}{s^2 + \alpha s + 1} \qquad (13)$$

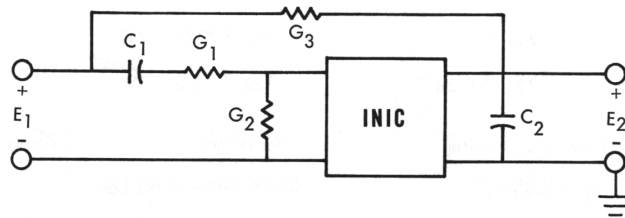A network configuration that realizes this voltage transfer function is shown in Figure 5-8.

Figure 5-8. INIC Low Pass Active Filter

The elements C_2 and G_3 form the A network of Figure 5-7. For this network we have the following y_{12} and y_{22} parameters:

$$y_{12a} = -G_3$$
$$y_{22a} = sC_2 + G_3 \qquad (14)$$

The elements C1, G1, and G2 shown in the figure form network B. This network has the y parameters:

$$y_{12b} = \frac{-sC_1G_1}{sC_1 + G_1}$$

$$y_{22b} = \frac{sC_1(G_1 + G_2) + G_1G_2}{sC_1 + G_1}$$

(15)

Substituting the expressions from equations (14) and (15) into equation (12), and re-arranging terms, we find that the voltage transfer for the circuit shown in Figure 5-8 is:

$$\frac{E_2}{E_1} = \frac{sC_1(G_3 - KG_1) + G_1G_3}{s^2C_1C_2 + s[C_2G_1 - KC_1G_2 + C_1(G_3 - KG_1)] + G_1(G_3 - KG_2)}$$

(16)

If we constrain the elements of this network so that G_3 equals KG_1, then equation (16) reduces to:

$$\frac{E_2}{E_1} = \frac{G_1G_3}{s^2C_1C_2 + s(C_2G_1 - KC_1G_2) + G_1(G_3 - KG_2)}$$

(17)

If we compare this equation with equation (13) we see that the constant H is positive; no signal inversion is provided by this circuit. Equations relating the values of the network elements to the constants α and H of equation (13) have been tabulated and, together with other design information on this circuit, they are presented in the circuits section of this handbook as Circuit No. 10.

In a similar fashion we may use passive RC networks and an INIC to realize a high pass filter. Such a network will have a transfer function of the form:

$$\frac{E_2}{E_1} = \frac{s^2H}{s^2 + \alpha s + 1}$$

(18)

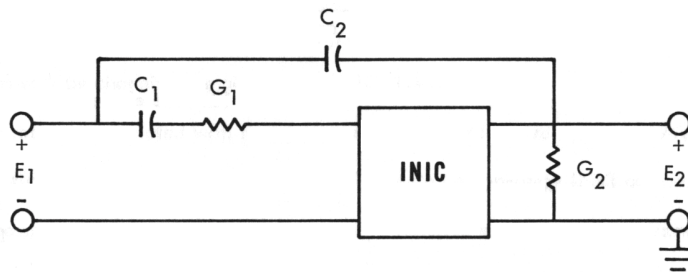A network configuration that realizes this function is shown in Figure 5-9.

Figure 5-9. INIC High Pass Active Filter

The elements C_2 and G_2 shown in this figure constitute network A of Figure 5-7. The y parameters for this network are:

$$y_{12a} = -sC_2$$

(19)

$$y_{22a} = sC_2 + G_2$$

The elements C_1 and G_1 constitute network B. Its y parameters are:

$$y_{22b} = -y_{12b} = \frac{sC_1 G_1}{sC_1 + G_1} \qquad (20)$$

If we substitute the expressions from equations (19) and (20) into equation (12), after rearranging terms, we find that the voltage transfer function for the overall network is

$$\frac{E_2}{E_1} = \frac{s^2 C_1 C_2 + sG_1(C_2 - KC_1)}{s^2 C_1 C_2 + s[C_1 G_2 + G_1(C_2 - KC_1)] + G_1 G_2} \qquad (21)$$

If we constrain the elements of this network so that C_2 equals KC_1, then equation (21) reduces to:

$$\frac{E_2}{E_1} = \frac{s^2 C_1 C_2}{s^2 C_1 C_2 + sC_1 G_2 + G_1 G_2} \qquad (22)$$

We see that the constant H of equation (18) is positive. The design information for this circuit is given in the circuits section of the handbook as Circuit No. 11.

As a final example of circuit design using an INIC, consider the band pass network with a frequency normalized voltage transfer function of the form:

$$\frac{E_2}{E_1} = \frac{-sH}{s^2 + \alpha s + 1} \qquad (23)$$

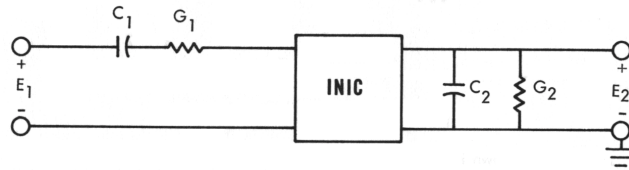A circuit that realizes this function is shown in Figure 5-10.

Figure 5-10. INIC Band Pass Active Filter

The elements C_2 and G_2 comprise network A of Figure 5-7. The y parameters are:

$$y_{12a} = 0$$
$$y_{22a} = sC_2 + G_2 \qquad (24)$$

The elements C1 and G1 comprise network B. Its y parameters are:

$$y_{22b} = -y_{12b} = \frac{sC_1 G_1}{sC_1 + G_1} \qquad (25)$$

The overall voltage transfer function for the circuit may be found by inserting the relations of (24) and (25) into equation (12). After rearranging terms we obtain:

$$\frac{E_2}{E_1} = \frac{-sKC_1 G_1}{s^2 C_1 C_2 + s(C_1 G_2 + C_2 G_1 - KC_1 G_1) + G_1 G_2} \qquad (26)$$

Note that the constant H of equation (23) is positive for this circuit, and that the circuit provides a signal inversion.

The voltage transfer function given in equation (26) points out some of the interesting properties that the INIC realization of the band pass network possesses. In the denominator, it should be noted that the INIC gain constant appears only in the coefficient of the first-degree term. Thus, in high Q realizations, it is possible to adjust the real part of the pole positions, i.e., the Q of the circuit, without significantly changing the magnitude of the pole positions, i.e., without significantly changing the resonant frequency. At a given frequency, therefore, we can control the bandwidth of the network by changing the gain of the INIC, i.e., by varying the values of either of the resistors associated with the operational amplifier INIC realization. A second interesting feature of this network is that by choosing a non-unity INIC gain constant, it is possible to design the network so that both resistive elements have the same value, and both capacitive elements have the same value. Thus, the problem of obtaining accurately specified passive element values is considerably minimized. Finally, if both resistors have the same value and are varied the same amount, the resonant frequency of the network is changed, although the Q of the network remains invariant. The same is true if the values of the capacitors are changed in the indicated manner. Thus we have a circuit where a single resistor may be changed to vary the bandwidth, and a pair of resistors may be changed to vary the resonant frequency, and the two effects do not interact. The design equations and other information for this circuit are given in the circuits section of the handbook as Circuit No. 12.

Conclusions

In this chapter, an entirely new "breed" of circuits has been presented, namely, circuits that use an INIC as the active element. The INIC can, of course, also be used to produce single negative-valued elements which in turn con be used to compensate for dissipation, to reduce input capacitance, etc.

We have used the INIC as an integral portion of circuits realizing low pass, high pass, and band pass voltage transfer functions. The band pass circuit is an especially attractive one because of the minimum number of elements used, the fact that all elements of a given kind have the some value, and the ease with which the Q and the resonant frequency of the network may be adjusted.

It should be noted that all the circuits given in this chapter share a common disadvantage; namely, their output impedance is not zero. Therefore, if such networks are cascaded, suitable isolating stages must be used to separate them. The advantage of ease of adjustment provided by some of these networks must therefore be weighed against the disadvantage of the requirement for the extra circuitry involved in isolating the filtering stages. A more detailed comparison of the advantages and disadvantages of the various realization schemes that have been presented in Chapters 2 through 5 are given in Chapter 7.

CHAPTER 6

Another Active Device: The Gyrator

In addition to the active elements that have been introduced in the preceding chapters, there is another one that deserves mention. It is called a gyrator. In this chapter we shall give a short introduction to the properties and potential uses of this element.

Definition of a Gyrator

A gyrator is a non-reciprocal two-port device defined by the equations:

$$I_1 = GE_2 \qquad (1)$$
$$I_2 = -GE_1$$

This two-port device is usually considered to have a common ground, and, in this case, the gyrator is represented by the symbol shown in Figure 6-1.
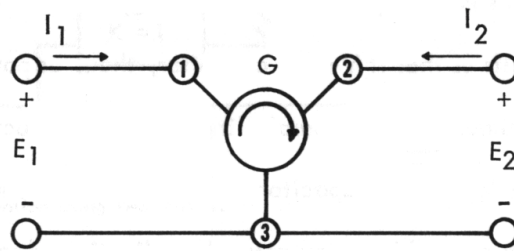
Figure 6-1. Gyrator Symbol

The constant G is called the gyration conductance. The reference arrow drawn inside the circle in the figure indicates that the gyration action from terminal 1 to terminal 2 with terminal 3 common (as shown) is the same as that which would occur from terminal 2 to terminal 3 if terminal 1 was used as the common terminal. Similarly, it is the same as that which would occur from terminal 3 to terminal 1 if terminal 2 were used as the common terminal. We shall see what this gyration action consists of in the paragraphs that follow.

Properties of the Gyrator

From equations (1) we see that the y parameters of a gyrator are:

$$y_{11} = 0 \qquad (2)$$
$$y_{12} = G$$
$$y_{21} = -G$$
$$y_{22} = 0$$

If we calculate the input admittance Y_{IN} of a two-port network, defined by its y parameters, when an admittance Y_2 is connected across the terminals of port 2, as shown in Figure 6-2:
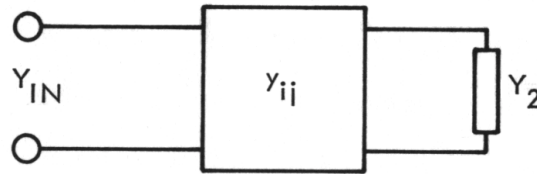
Figure 6-2. The Input Admittance of a Terminated Two-Port Network

we obtain:

$$Y_{IN} = y_{11} \frac{y_{12}y_{21}}{y_{22} + Y_2} \qquad (3)$$

For the case where the two-port is a gyrator we may substitute the y parameters from equations (2) into the expression in equation (3). Thus we obtain:

$$Y_{IN} = \frac{G^2}{Y_2} \qquad (4)$$

This is the input admittance of a gyrator terminated in admittance Y_2. Equation (4) tells us that if Y_2 is a capacitor, then, at part 1 of the gyrator we see a two-terminal behavior exactly paralleling that of an inductor. In other words, a gyrator can be used to "gyrate" a capacitor into an inductor. Thus, with resistors, capacitors, and gyrators, we can achieve any network realization that can be achieved with resistors, capacitors, and inductors. If we can obtain a gyrator with a small value of G, then we can gyrate very small capacitors into very large inductors, a most useful feat!

One other property of the gyrator may be of interest. This concerns the relationships at the terminals of the gyrator. For any two-port device, the total instantaneous power consumed by the device is:

$$p(t) = e_1(t)i_1(t) + e_2(t)i_2(t) \qquad (5)$$

Substituting relations equivalent to those given in equation (1), but in terms of functions of time, into the above equation, we see that

$$p(t) = e_1(t)i_1(t) - e_1(t)i_1(t) = 0 \qquad (6)$$

Thus we see that the gyrator neither adds energy to the circuit in which it is used, nor consumes it. As such, its terminal properties are those of a lossless passive network component. We shall see, however, that its realization inevitably requires the use of active elements.

A Gyrator Realization

There are several ways of realizing a gyrator. One of the methods uses two INICs. Consider the circuit shown in Figure 6-3.
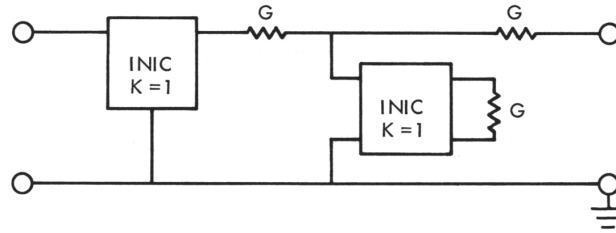
Figure 6-3. Gyrator Realization Using Two INIC's

It is easily shown that this circuit has the y parameters given in equation (2), and thus functions as a gyrator. The values of the resistors (in mhos) shown in the figure determine the value of the gyration conductance. References to some other methods for realizing gyrators are given in Appendix A.

Circuit Realizations

There are considerably fewer results available in the literature regarding the use of gyrators and RC circuits for the realization of transfer functions than there are for any of the other classes of networks that have been discussed in this handbook. Since the state of the art is relatively new, and since the active elements are considerably more complicated than any which have been discussed in the previous chapters, we shall not present general circuits for the realization of low pass, high pass and band pass circuits as was done in those chapters. To give an example of one form that such realizations may take, however, consider the circuit shown in Figure 6-4.
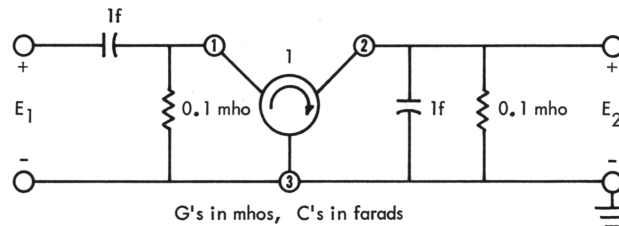
Figure 6-4. Gyrator Band Pass Active Filter

The voltage transfer function for this circuit is:

$$\frac{E_2}{E_1} = \frac{s}{s^2 + 0.2s + 1.01} \qquad (7)$$

Thus, the circuit realizes a band pass voltage transfer function.

Conclusions

Due to the complicated nature of the realizations for the gyrator, this network element has not achieved a wide usage at this time. In addition to the disadvantage of complexity, realizations that use it as the active element have two other disadvantages in that their output impedance will not be zero, and they will be capable only of the gain that a passive RLC circuit is capable of. Despite these disadvantages, the gyrator also has some potential advantages. First, its lossless nature provides a theoretical bar to circuit instability, since if no power is being supplied to the circuit, instability cannot occur. Second, since capacitors in general have a higher quality factor (lower dissipation) than inductors, gyration of a capacitor may produce a better inductor than those that are readily available. Finally, the possibility of using a gyrator for impedance multiplication implies the ability to realize very low frequency circuits without the need of relatively large-valued reactive elements.

CHAPTER 7

A Summary

In Chapters 2 through 5 of this handbook four different active RC synthesis techniques were presented. These were the infinite-gain single-feedback technique, the infinite-gain multiple-feedback technique, a technique using controlled sources, and a technique using negative-immittance converters. The various methods were all applied to the realization of low pass, high pass, and band pass voltage transfer functions. At this point the reader may well ask, "If I want to realize a low pass function (or a high pass or a band pass one) which of the methods is best?' The answer to the question, of course, depends on how the word "best" is defined If "best" means fewest elements, then the infinite-gain single feedback technique is certainly eliminated, If the network is to be cascaded with other networks, then the NIC approach is probably not a good one. Thus, the answer to such a question depends on the details of the application, which will vary considerably from one situation to another.

The purpose of this handbook has been to give the prospective user several different techniques for each of the filter realizations, in order to permit him flexibility in selecting the technique that more nearly meets his specific application. To provide a further guide to such a choice, some of the advantages and disadvantages of the various realization techniques are summarized in Table I. A study of this table will provide a good review of the material that has gone before.

Table 1.   Summary of the Advantages and Disadvantages of the Various Realization Techniques

Property	Realization Technique			
	Infinite-Gain Single-Feedback	Infinite-Gain Multiple-Feedback	Controlled Source	Negative-Immittance Converter
Minimal number of Network elements	-	+	+	+
Ease of adjustment Of characteristics	-	0	0	+
Stability of characteristics	+	+	-	-
Low output Impedance	+	+	+	-
Presence of Summing input	+	-	-	-
Relatively high Gain available	+	-	+	+
Low spread of Element values	+	-	+	+
High-Q realization possible	+	-	+	+
+ Indicates the realization is superior for the indicated property				
0 Indicates the realization is average for the indicated property				
- Indicates the realization is inferior for the indicated property				

This concludes our introduction to the wonderful world of active RC networks, a world without inductors. It is hoped that the reader will find the techniques that have been presented in this handbook useful, and that he will be able to apply them to his own filtering problems. Needless to say, the Applications Engineering Department of Texas Instruments welcomes any questions or comments that the reader might have on the material of this booklet or on any other operational amplifier application. Feel free to call us at any time.

SECTION II

Circuits

Introduction

All of the examples shown in this section have been proven on the bench with the results shown. In addition, the transfer functions are given along with methods for determining component values and comments on the nominal range of values. We trust that these "hints and kinks" will enable you to readily modify the circuits given for your application.

Two possible points of confusion, however, deserve mention. In the theory section we have, for simplicity, worked with frequency normalized transfer functions $\omega_o \left(= 2\pi f_o \right)$ and conductance $G \left(= 1/R \right)$. In moving on to the real world of circuits, we find it convenient to "un-normalize" the transfer functions $f_o \left(= \omega_o / 2\pi \right)$ and deal with resistance $R \left(= 1/G \right)$. Have made this shift explicit, we trust it will not create problems.

Please note:

- Since the design formulas give correct values to within an arbitrary constant, an impedance denormalization constant, k, is included and is to be chosen for convenience.

- The original edition of this handbook had part numbers of op amps listed on the schematics. While these part numbers were meaningful in 1966 when the original edition went to press – they are meaningless today.

- The state of the art in op amps has progressed, making these relatively low frequency examples somewhat dated. They are left "as-is", because they illustrate circuit design techniques – not state of the art performance levels.

It is our hope that the circuits presented will trigger the idea that develops into your circuit. Sharing your circuit with us will enable us to share it with other engineers. Similarly, we would welcome the opportunity to share our experience and the latest advancements in network theory and amplifiers with you.

TEXAS
INSTRUMENTS

CIRCUIT 1: Single Feedback Low Pass

Network B

$$\frac{E_2}{E_1} = \frac{-H\omega_o{}^2}{s^2 + \alpha\omega_o s + \omega_o{}^2}$$

$$\omega_o = 2\pi f_o$$

$$A_o = H$$

Amplifier Open Loop Response

Overall Transfer Function Response
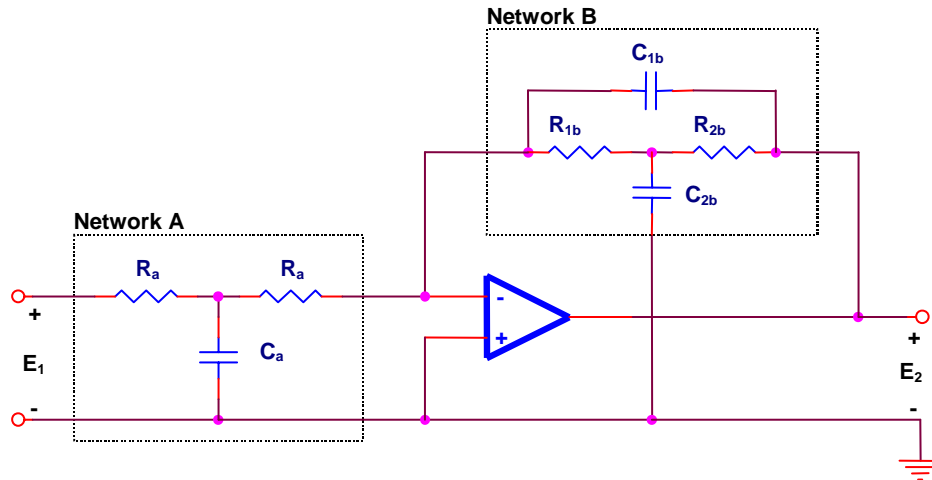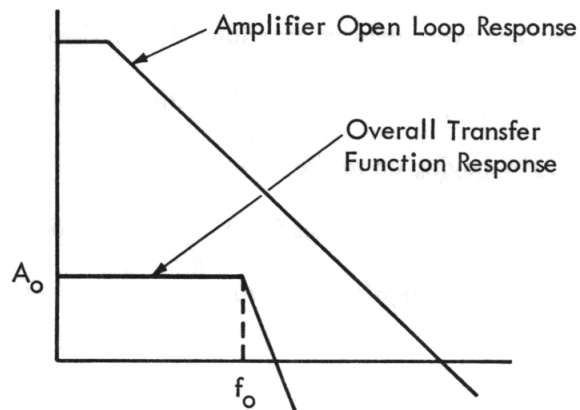
Choose :

α (= $\sqrt{2}$ for "maximally flat" 'Butterworth' response

Let :

$b = (2.5 - \alpha)$

Calculate :

$$C_a = \frac{4H}{\alpha^2} \frac{k}{2\pi f_o}$$

$$C_{1b} = \frac{k}{2\pi f_o}$$

$$C_{2b} = \frac{b^2}{\alpha b - 1} \cdot \frac{k}{2\pi f_o}$$

$$R_a = \frac{\alpha}{2Hk} \quad \text{(two such resistors)}$$

$$R_{1b} = \frac{1}{b} \cdot \frac{1}{k}$$

$$R_{2b} = \left(\alpha - \frac{1}{b}\right) \cdot \frac{1}{k}$$
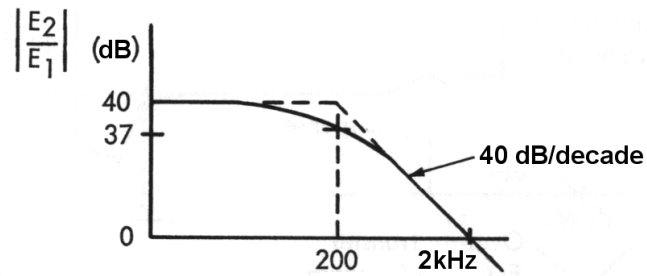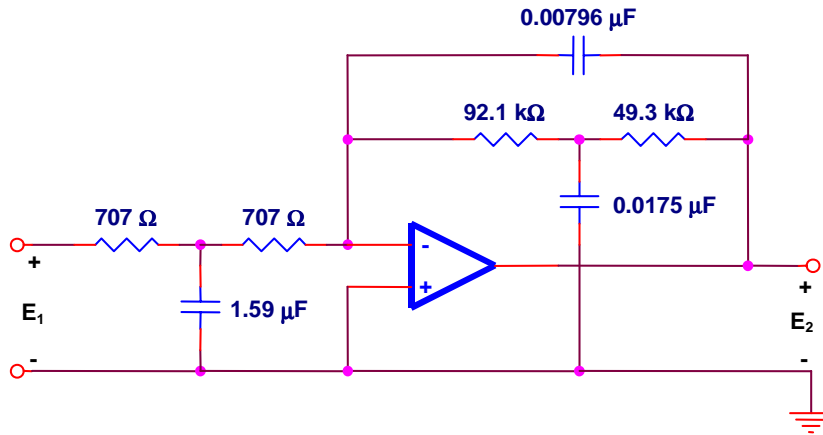
EXAMPLE

Want :

$$f_o = 200 \text{ Hz}$$

$$|A_o| = 100 \ (40\text{dB})$$

$$\alpha = \sqrt{2}$$

$$k = 10^{-5}$$

Calculate :

$$C_a = 1.59 \ \mu F$$

$$C_{1b} = 0.00796 \ \mu F$$

$$C_{2b} = 0.0175 \ \mu F$$

$$R_a(2) = 707 \ \Omega$$

$$R_{1b} = 92.1 \ k\Omega$$

$$R_{2b} = 49.3 \ k\Omega$$

CIRCUIT 2: Single Feedback High Pass

$$\frac{E_2}{E_1} = \frac{-Hs^2}{s^2 + \alpha\omega_o s + \omega_o^{\;2}} \qquad \begin{array}{l} \omega_o = 2\pi f_o \\[4pt] A_o = H \end{array}$$
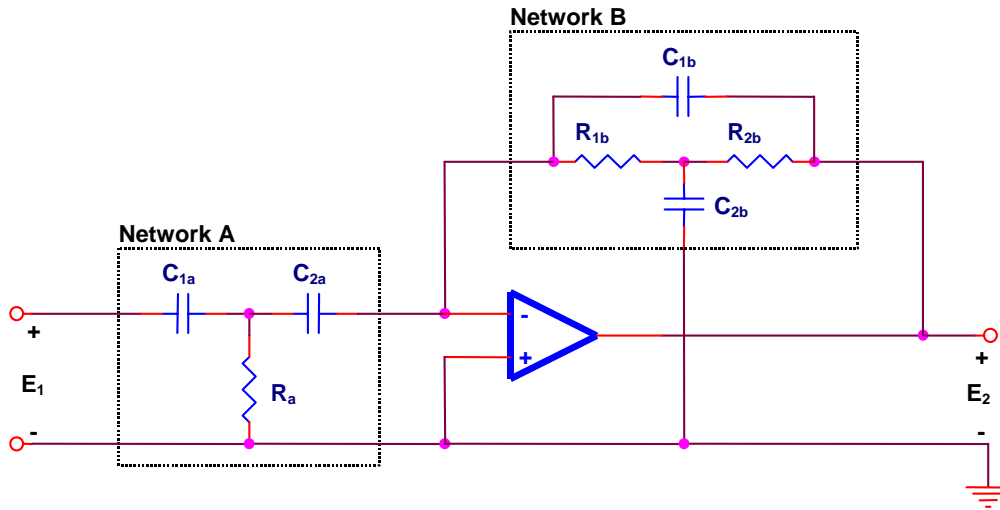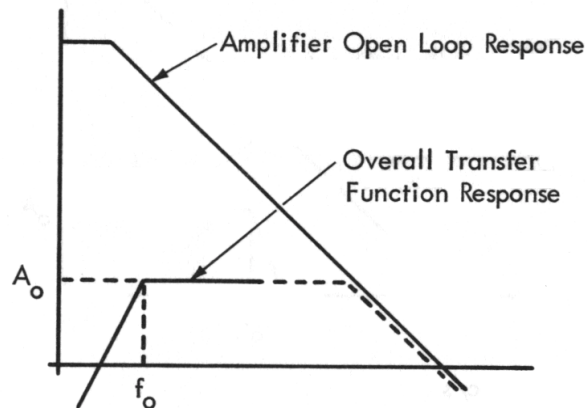
Choose :

$\alpha \;(=\sqrt{2}$ for "maximally flat" response, 40 dB/decade rolloff)

$\alpha > \sqrt{2}$, slower rolloff

$\alpha < \sqrt{2}$, peaking, faster rolloff

Let :

$b = (2.5 - \alpha)$

Calculate :

$$C_{1a} = C_{2a} = 2H\frac{k}{2\pi f_o} \qquad R_a = \frac{1}{4H\alpha} \cdot \frac{1}{k}$$

$$C_{1b} = \frac{k}{2\pi f_o} \qquad\qquad R_{1b} = \frac{1}{b} \cdot \frac{1}{k}$$

$$C_{2b} = \frac{b^2}{\alpha b - 1} \frac{k}{2\pi f_o} \qquad R_{2b} = \left(\alpha - \frac{1}{b}\right) \cdot \frac{1}{k}$$
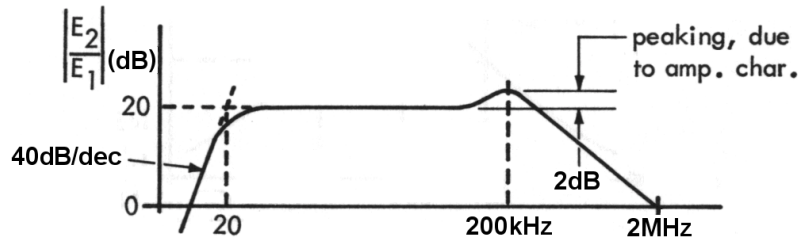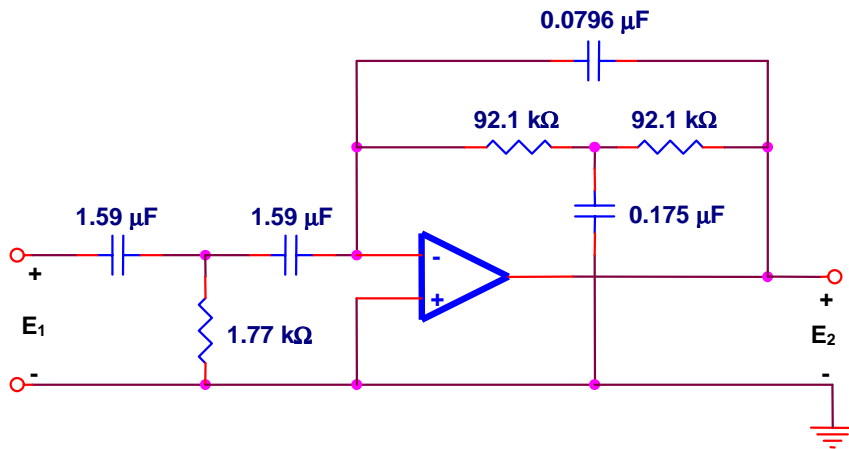
EXAMPLE

Want :

$f_o = 20\text{Hz}$

$|A_o| = 10 \ (20 \text{ dB})$

$\alpha = \sqrt{2}$

Choose :

$k = 10^{-5}$

Calculate :

$C_{1a} = C_{2a} = 1.592 \ \mu\text{F}$

$C_{1b} = 0.0796 \ \mu\text{F}$

$R_a = 1.77 \ \text{k}\Omega$

$R_{1b} = 92.1 \ \text{k}\Omega$

$R_{2b} = 49.3 \ \text{k}\Omega$

TEXAS INSTRUMENTS

CIRCUIT 3: Single Feedback Band Pass

Network B

R_{1b} R_{2b}

C_{1b} C_{2b}

R_{3b} C_{3b}

Network A

R_a C_a

+

E_1

-

-

+

+

E_2

-

$$\frac{E_2}{E_1} = \frac{-H\omega_o s}{s^2 + \alpha\omega_o s + \omega_o^{\,2}}$$

$$\omega_o = 2\pi f_o$$

$$A_o = H/\alpha$$

$$Q = 1/\alpha$$
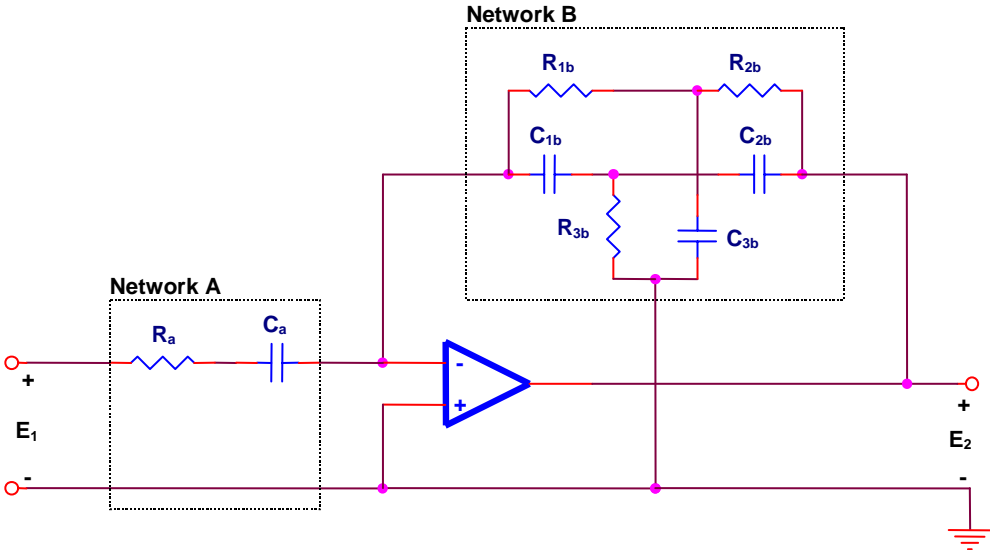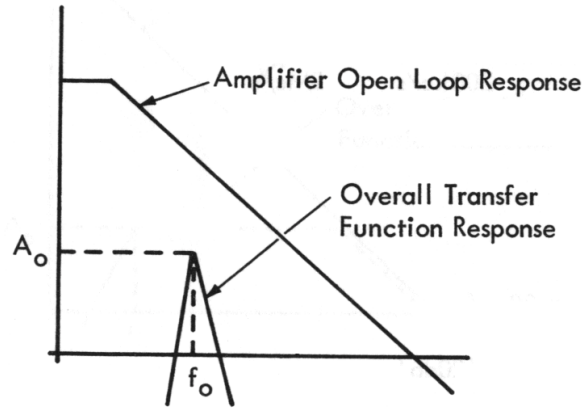
Calculate :

Let :

$$\alpha = 1/Q$$

$$b = (2.5 - \alpha)\frac{1+\alpha}{2+\alpha}$$

$$H = A_o/Q$$

$$C_a = H\frac{k}{2\pi f_o}$$

$$C_{1b} = b\frac{k}{2\pi f_o}$$

$$C_{2b} = \frac{b}{b-1} \cdot \frac{k}{2\pi f_o}$$

$$C_{3b} = \frac{b^2}{(b-1)(1+\alpha)} \cdot \frac{k}{2\pi f_o}$$

$$R_a = \frac{1}{H} \cdot \frac{1}{k}$$

$$R_{1b} = \frac{1}{b} \cdot \frac{1}{k}$$

$$R_{2b} = \frac{(b-1)}{b} \cdot \frac{1}{k}$$

$$R_{3b} = \frac{(b-1)(\alpha+1)}{b^2} \cdot \frac{1}{k}$$

TEXAS
INSTRUMENTS

7.96 kΩ   **2.04 kΩ**

0.200 µF   **0.778 µF**

0.888 µF

1.79 kΩ

10 kΩ   **0.159 µF**

E₁   E₂
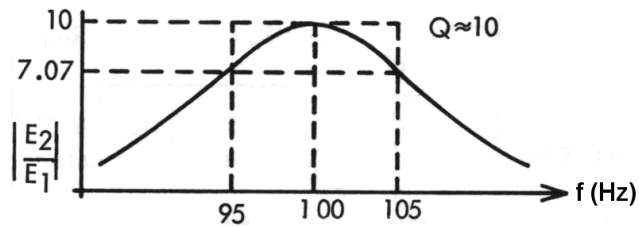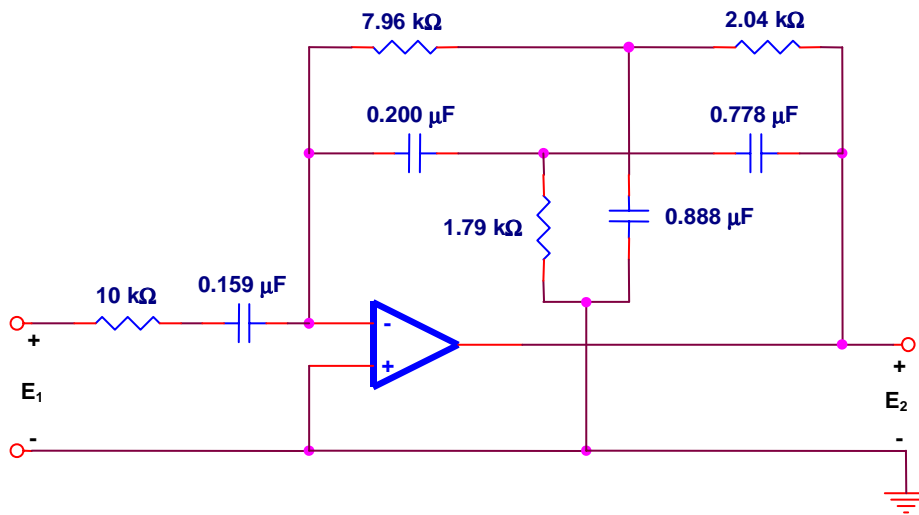
EXAMPLE

Want :

$f_o = 100 \text{ Hz}$

$|A_o| = 10 \ (20 \text{ dB})$

$Q = 10$

Choose :

$k = 10^{-4}$

Calculate :

$C_a = 0.159 \ \mu F$

$C_{1b} = 0.200 \ \mu F$

$C_{2b} = 0.778 \ \mu F$

$C_{3b} = 0.888 \ \mu F$

$R_a = 10 \ k\Omega$

$R_{1b} = 7.96 \ k\Omega$

$R_{2b} = 2.04 \ k\Omega$

$R_{3b} = 1.79 \ k\Omega$

CIRCUIT 4: Multiple Feedback Low Pass

$$\frac{E_2}{E_1} = \frac{-H\omega_o^2}{s^2 + \alpha\omega_o s + \omega_o^2}$$

$$A_o = H$$

$$\omega_o = 2\pi f_o$$
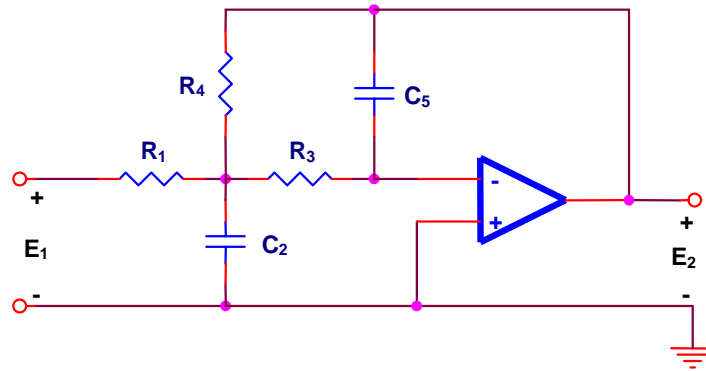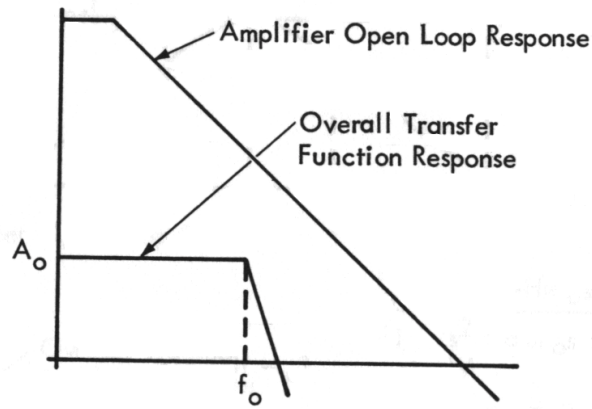
TRANSFER FUNCTION

$$\frac{E_2}{E_1} = \frac{-1/R_1 R_3}{s^2 C_2 C_5 + sC_5\left(1/R_1 + 1/R_3 + 1/R_4\right) + 1/R_3 R_4}$$

Choose :

$$C = \frac{k}{2\pi f_o}, \text{ (defines k)}$$

$$H = |A_o|$$

$\alpha \ (= \sqrt{2} \text{ for "max imally flat"}$
response, 40 dB / decade rolloff)

Calculate :

$$C_5 = C = \frac{k}{2\pi f_o}$$

$$C_2 = \frac{4}{\alpha^2} (H+1) \frac{k}{2\pi f_o}$$

$$R_1 = \frac{\alpha}{2Hk}$$

$$R_3 = \frac{\alpha}{2(H+1)k}$$

$$R_4 = \frac{\alpha}{2k} = HR_1$$
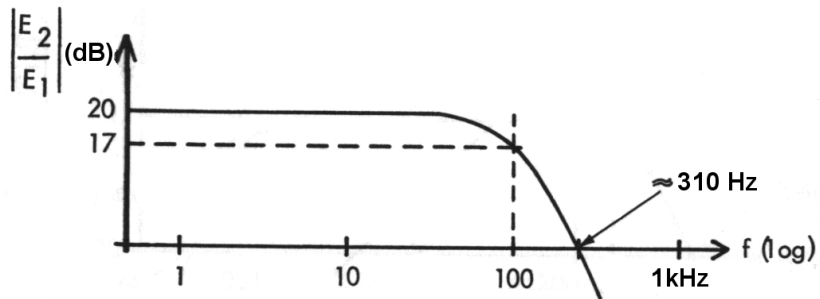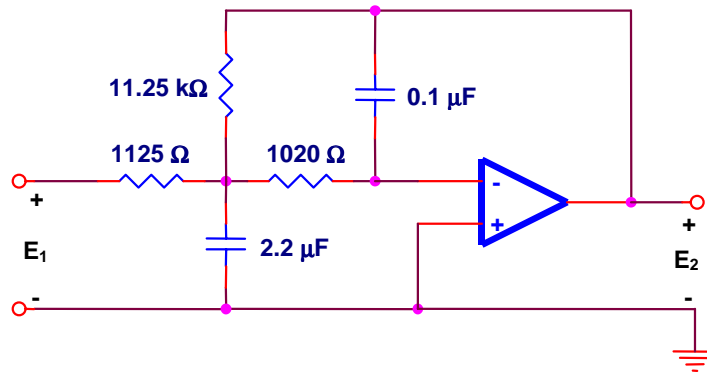
EXAMPLE

Want :

$f_o = 100$ Hz

$A_o = 10\ (20\ \text{dB})$

$\alpha = \sqrt{2}$

Choose :

$C = 0.1\ \mu F$

Calculate :

$k = 6.28 \times 10^{-5}$

$C_5 = 0.1\ \mu F$

$C_2 = 2.2\ \mu F$

$R_1 = 1125\ \Omega$

$R_3 = 1020\ \Omega$

$R_4 = 11.25\ k\Omega$

TEXAS
INSTRUMENTS

CIRCUIT 5: Multiple Feedback High Pass

$$\frac{E_2}{E_1} = \frac{-Hs^2}{s^2 + \alpha\omega_o s + \omega_o^2}$$

$$\omega_o = 2\pi f_o$$
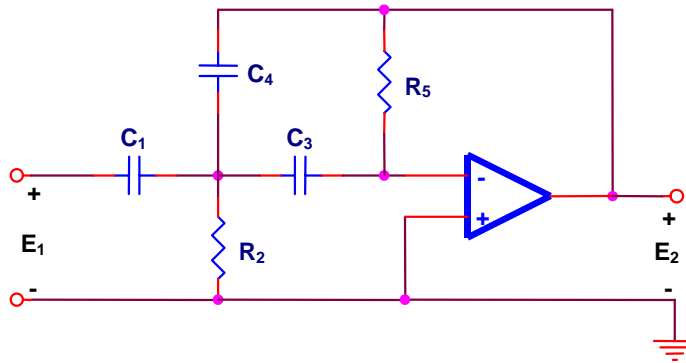
$$A_o = H$$
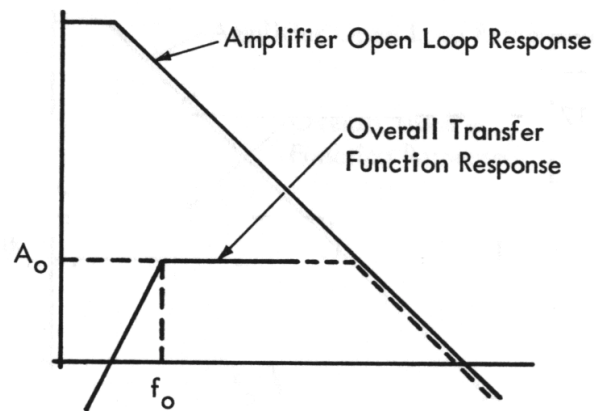
TRANSFER FUNCTION

$$\frac{E_2}{E_1} = \frac{-s^2 C_1 C_3}{s^2 C_3 C_4 + s(C_1 + C_3 + C_4)/R_5 + 1/R_2 R_5}$$

Choose :

$$C = \frac{k}{2\pi f_o}, \text{ (defines k)}$$

$$H = |A_o|$$

α (= $\sqrt{2}$ for "maximally flat"
response, 40 dB/decade rolloff)

Calculate :

$$C_1 = C_3 = \frac{k}{2\pi f_o} = C$$

$$C_4 = C/H$$

$$R_2 = \frac{\alpha}{k(2 + 1/H)}$$

$$R_5 = \frac{H(2 + 1/H)}{\alpha k}$$
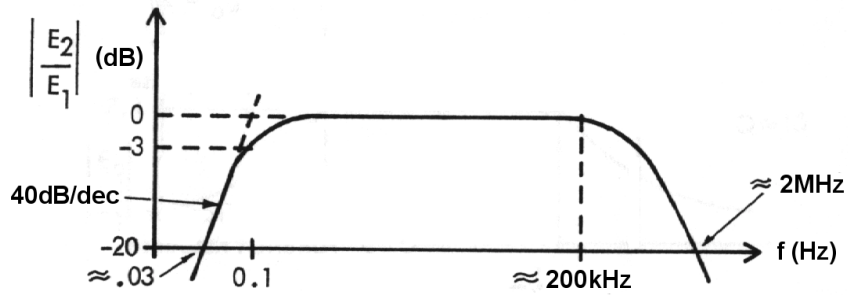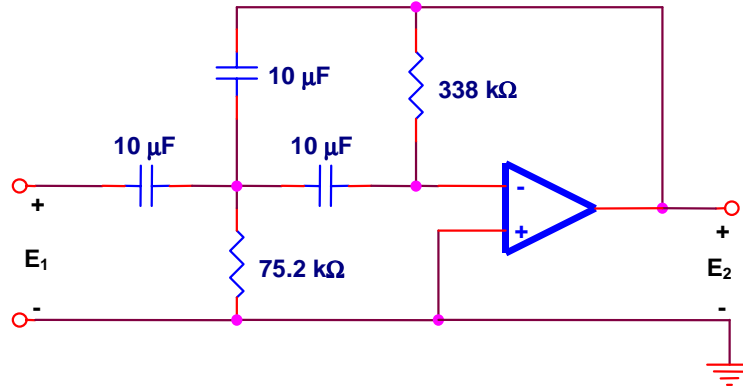
EXAMPLE

Want :

$f_o = 0.1$ Hz

$A_o = 1 \, (0 \text{ dB})$

$\alpha = \sqrt{2}$

Choose :

$C = 10 \, \mu F$

Calculate :

$k = 6.28 \times 10^{-6}$

$C_1 = C_3 = 10 \, \mu F$

$C_4 = 10 \, \mu F$

$R_2 = 75.2 \text{ k}\Omega$

$R_5 = 338 \text{ k}\Omega$

CIRCUIT 6: Multiple Feedback Band Pass

$$\frac{E_2}{E_1} = \frac{-H\omega_o s}{s^2 + \alpha\omega_o s + {\omega_o}^2}$$

$$A_o = H/\alpha$$

$$Q = 1/\alpha$$

$$\omega_o = 2\pi f_o$$
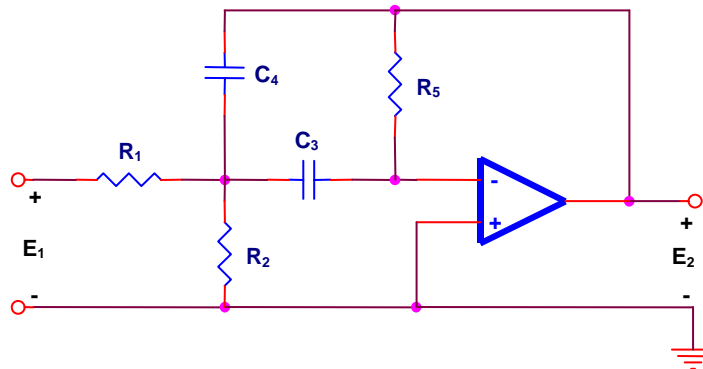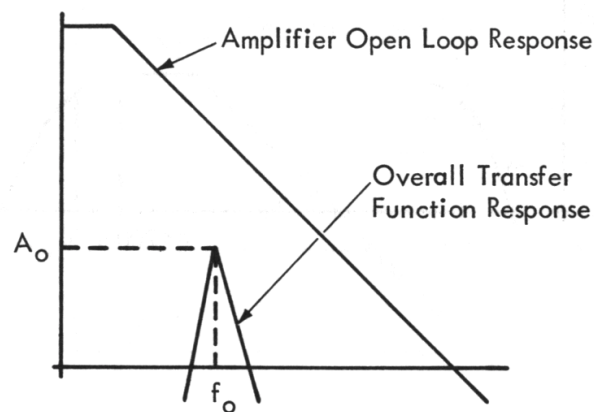
TRANSFER FUNCTION

$$\frac{E_2}{E_1} = \frac{-sC_3/R_1}{s^2 C_3 C_4 + s \cdot 1/R_5 (C_3 + C_4) + 1/R_5 (1/R_4 + 1/R_2)}$$

Choose :

$$C = \frac{k}{2\pi f_o}, \text{ (defines k)}$$

$$\alpha = 1/Q$$

$$H = |A_o|/Q$$

Calculate :

$$C_3 = C_4 = \frac{k}{2\pi f_o} = C$$

$$R_1 = 1/Hk$$

$$R_2 = \frac{1}{(2Q-H)k}$$

$$R_5 = \frac{2Q}{k}$$

10 μF

200 kΩ

10 kΩ

10 μF

E_1

527 Ω

E_2

$Q \approx 10$
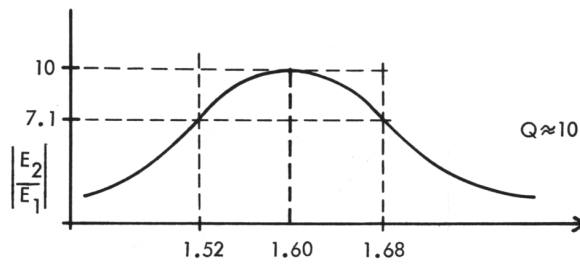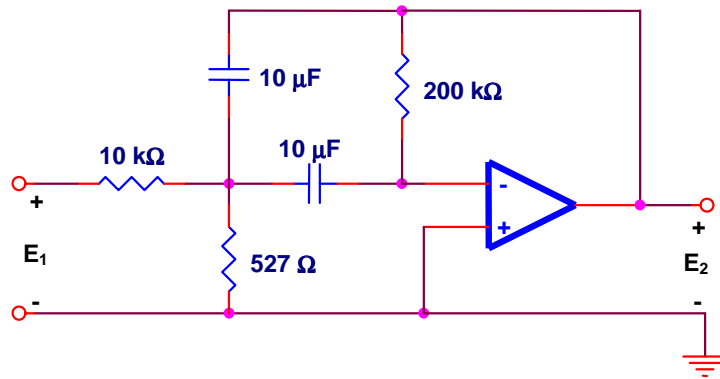
EXAMPLE

Want :

$f_o = 1.6$ Hz

$Q = 10$

$A_o = 10 \ (20 \text{ dB})$

Choose :

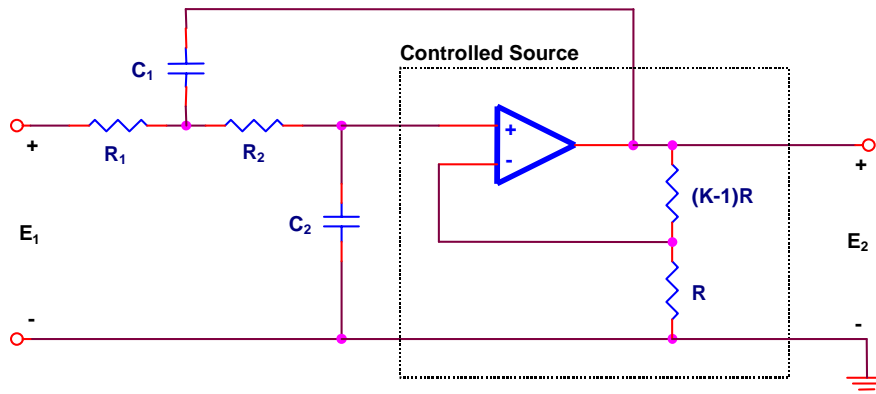$C = 10 \ \mu F$

Calculate :

$k = 10^{-4}$

$C_3 = C_4 = 10 \ \mu F$

$H = 1$

$R_1 = 10 \text{ k}\Omega$

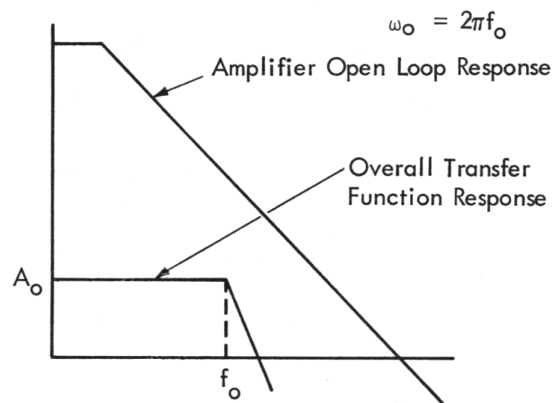$R_2 = 527 \ \Omega$

$R_5 = 200 \text{ k}\Omega$

CIRCUIT 7: Controlled Source Low Pass

$$\frac{E_2}{E_1} = \frac{H\omega_o{}^2}{s^2 + \alpha\omega_o s + \omega_o{}^2}$$

$$A_o = H = K$$

$$\omega_o = 2\pi f_o$$

TRANSFER FUNCTION

$$\frac{E_2}{E_1} = \frac{K/R_1R_2}{s^2C_1C_2 + s[C_2/R_1 + C_2/R_2 + C_1/R_2(1-K)] + 1/R_1R_2}$$

Choose :

$C_1 = \dfrac{k}{2\pi f_o}$ defines k

$K = A_o$

α ($= \sqrt{2}$ for "maximally
flat" response)

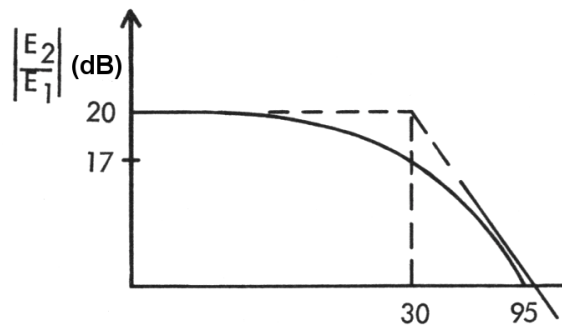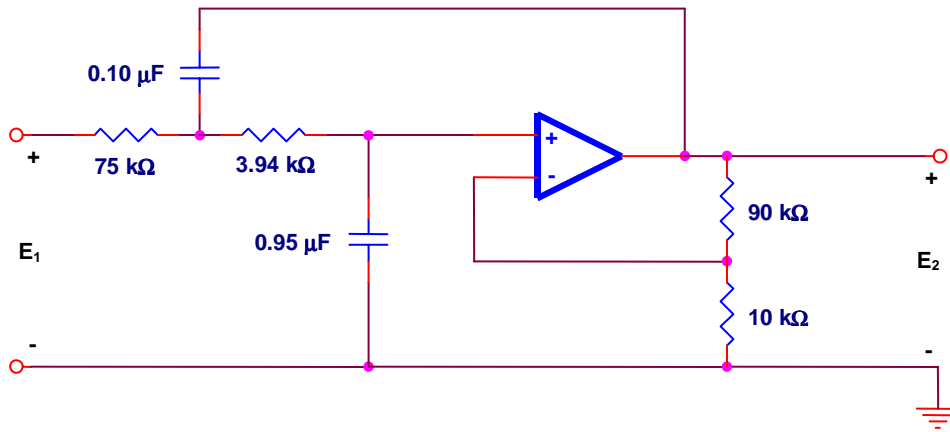R and $(K-1)R$ for appropriate
speed and power consumption

Calculate :

$$m = \frac{a^2}{4} + (K-1)$$

$$C_2 = mC_1 = \frac{mk}{2\pi f_o}$$

$$R_1 = \frac{2}{\alpha k}$$

$$R_2 = \frac{\alpha}{2mk}$$

EXAMPLE

Want :

$f_o = 30$ Hz

$\alpha = \sqrt{2}$

$C_1 = 0.1$ μF

$A_o = 10 \left(20 \text{ dB}\right)$

Calculate :

$m = 9.5$

$k = 1.89 \times 10^{-5}$

$C_2 = 0.95$ μF

$R_1 = 75$ kΩ

$R_2 = 3.94$ kΩ

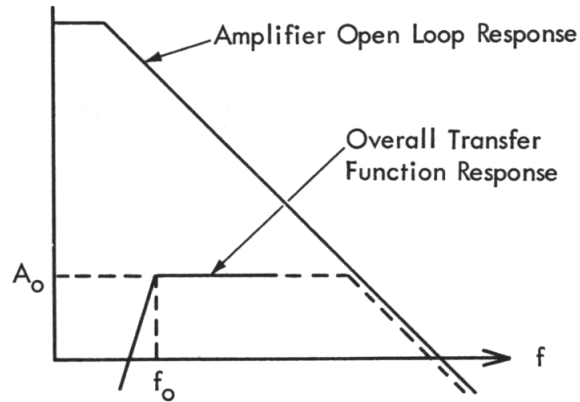$K = 10$

TEXAS INSTRUMENTS

CIRCUIT 8: Controlled Source High Pass

$$\frac{E_2}{E_1} = \frac{Hs^2}{s^2 + \alpha\omega_o s + \omega_o{}^2}$$

$$A_o = K$$

$$\omega_o = 2\pi f_o$$
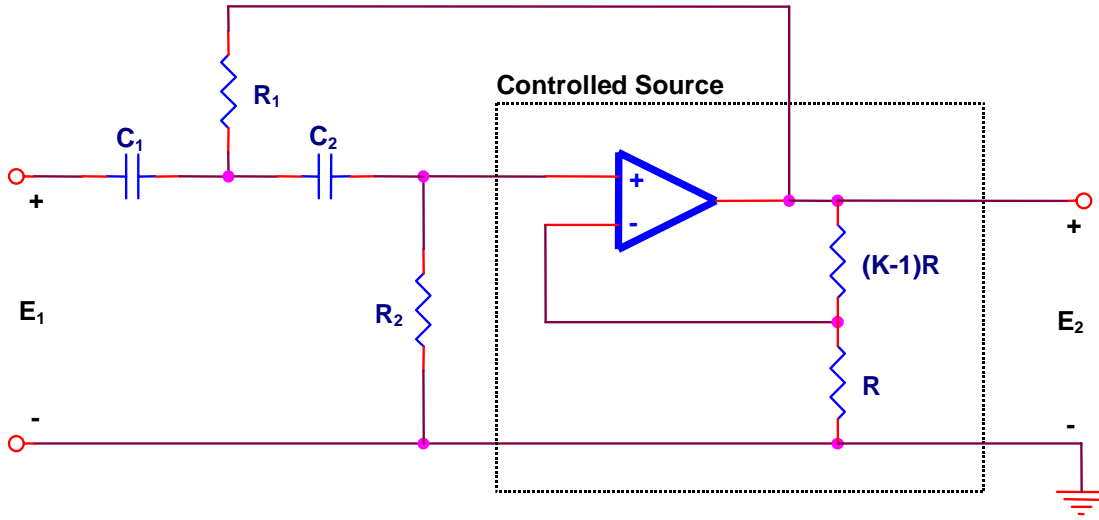
Choose :

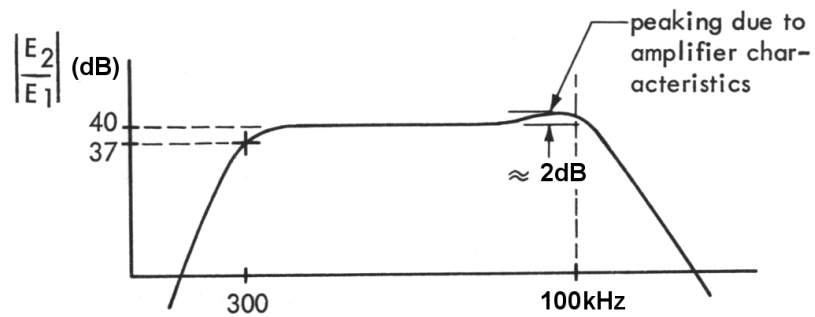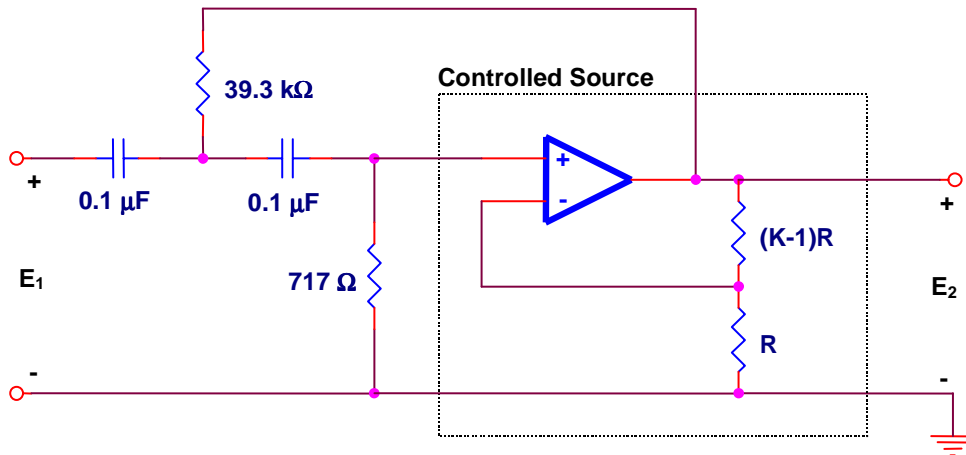$\alpha \left(= \sqrt{2} \text{ for "max imally flat", 'Butterworth' response} \right)$

$C_1 = \dfrac{k}{2\pi f_o} \left(\text{defines } k \right)$

$C_2 = C_1$

Calculate :

$R_1 = \dfrac{\alpha + \sqrt{\alpha^2 + 8(K-1)}}{4k}$

$R_2 = \dfrac{4}{\alpha + \sqrt{\alpha^2 = 8(K-1)}} \cdot \dfrac{1}{k}$

TEXAS INSTRUMENTS

EXAMPLE

Want :

$f_o = 300$ Hz

$\alpha = \sqrt{2}$
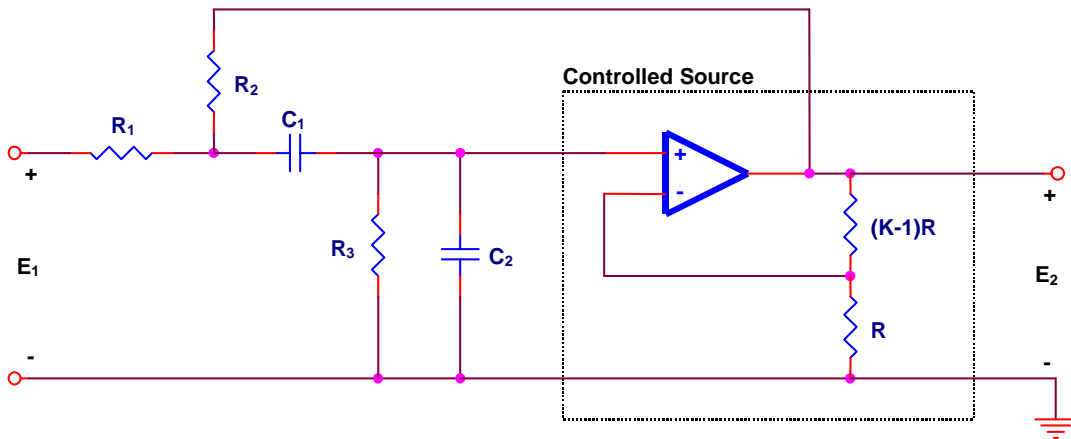
$A_o = 100 \ (40 \ dB)$

$C_1 = 0.1 \ \mu F = C_2$

Calculate :

$k = 1.884 \times 10^{-4}$

$R_1 = 39.3 \ k\Omega$

$R_2 = 717 \ \Omega$

$K = 100$

CIRCUIT 9: Controlled Source Band Pass

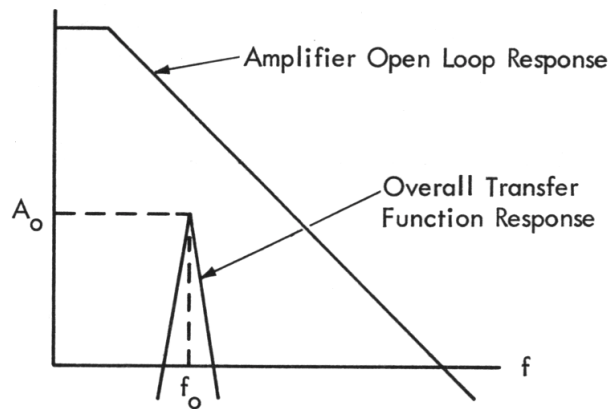$$\frac{E_2}{E_1} = \frac{H\omega_o s}{s^2 + \alpha\omega_o s + \omega_o^2}$$

$$A_o = H/\alpha \qquad H = K$$

$$Q = 1/\alpha \qquad \omega_o = 2\pi f_o$$

TRANSFER FUNCTION

$$\frac{E_2}{E_1} = \frac{Ks\dfrac{C_1}{R_1}}{s^2 C_1 C_2 + s\left[\dfrac{C_1}{R_3} + \dfrac{C_1 + C_2}{R_1} + \dfrac{C_2}{R_2} + \dfrac{C_1}{R_2}(1-K)\right] + \dfrac{1}{R_3}\cdot\left(\dfrac{R_1 + R_2}{R_1 R_2}\right)}$$

Choose :

$$C_1 = \frac{k}{2\pi f_o} \text{ (defines k)}$$

R and $(K-1R)$ for desired
op amp speed and power

Calculate :

$$C_2 = \frac{1}{2}C_1 = \frac{k}{4\pi f_o}$$
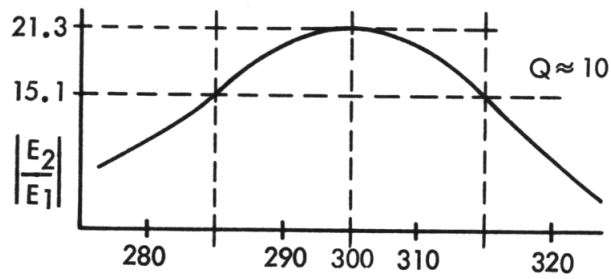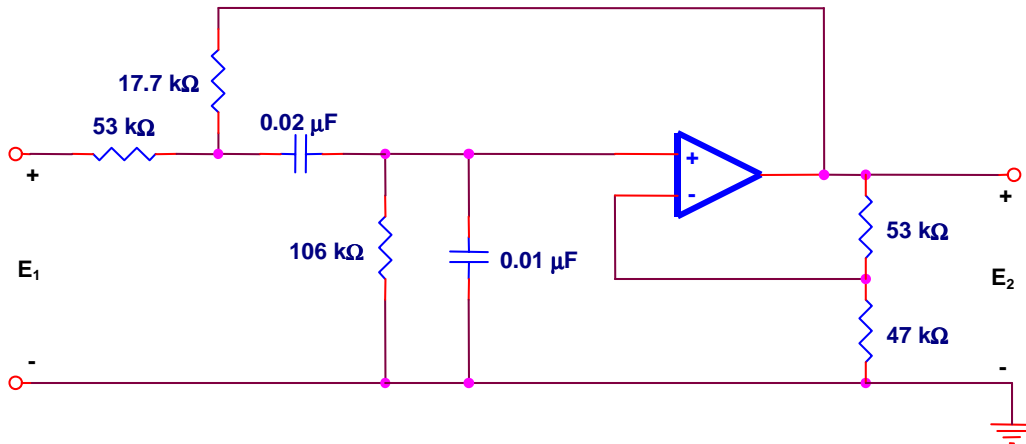
$$R_1 = \frac{2}{k}$$

$$R_2 = \frac{2}{3k} = \frac{R_1}{3}$$

$$R_3 = \frac{4}{k} = 2R_1$$

$$K = \frac{1}{3}\left(6.5 - \frac{1}{Q}\right)$$

TEXAS
INSTRUMENTS

EXAMPLE

Want :

$f_o = 300$ Hz

$Q = 10$

$C_1 = 0.02$ μF

Calculate :

$K = 2.133$

$A_o = 21.33$ (26.6 dB)

$C_1 = 0.02$ μF

$C_2 = 0.01$ μF

$k = 37.8 \times 10^{-6}$

$R_1 = 53$ kΩ

$R_2 = 17.7$ kΩ

$R_3 = 106$ kΩ

CIRCUIT 10: INIC Low Pass
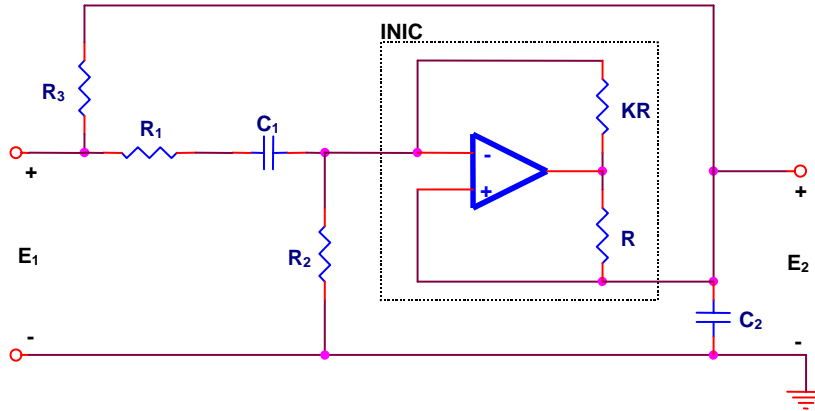
$$\frac{E_2}{E_1} = \frac{H\omega_o{}^2}{s^2 + \alpha\omega_o s + \omega_o{}^2} \qquad\qquad \omega_o = 2\pi f_o$$

$$\frac{E_2}{E_1} = \frac{sC_1(1/R_3 - K/R_1)^{\nearrow 0} + 1/R_1 R_3}{s^2 C_1 C_2 + s[C_2/R_1 - C_1/R_2 + C_1(1/R_3 - K/R_1)] + 1/R_1(1/R_3 - K/R_2)}$$
$$\underset{0}{}$$

Choose :

$\alpha\ (=\sqrt{2}$ for "maximally flat" response)

Let :

$K = 1$

$R_1 = R_3$

Calculate :

$$b = \frac{\alpha}{2} + \sqrt{\frac{\alpha^2}{4} + (A_o - 1)}$$

$$C_2 = \frac{k}{2\pi f_o} \ (\text{defines } k)$$

$$C_1 = \frac{A_o}{b^2} \cdot \frac{k}{2\pi f_o} = \frac{A_o}{b^2} C_2$$

$$R_1 = R_3 = \frac{b}{A_o k}$$

$$R_2 = \frac{b}{(A_o - k)k}$$

15.1 kΩ

15.1 kΩ

0.696 µF

(10 kΩ)

VCC

E_1

16.8 kΩ

(10 kΩ)

VCC

E_2

1.0 µF

20

17

$\left|\dfrac{E_2}{E_1}\right|$,dB

40dB/dec

f (Hz)

4

≈ 12.7
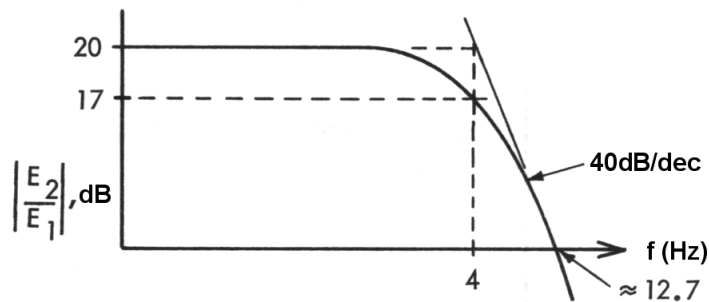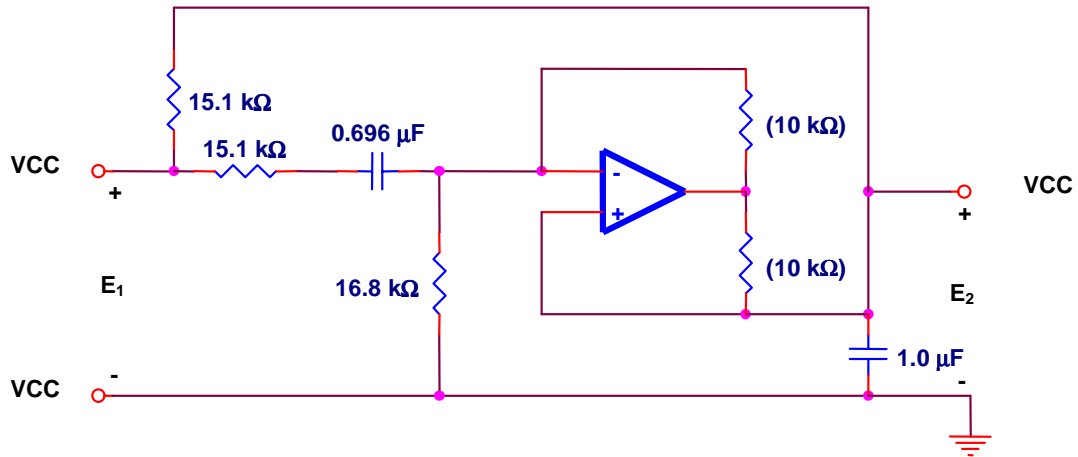
EXAMPLE

Want :

$f_o = 4 \text{ Hz}$

$\alpha = \sqrt{2}$

$A_o = 10 \ (20 \text{ dB})$

$C_2 = 1 \ \mu F$

Calculate :

$b = 3.79$
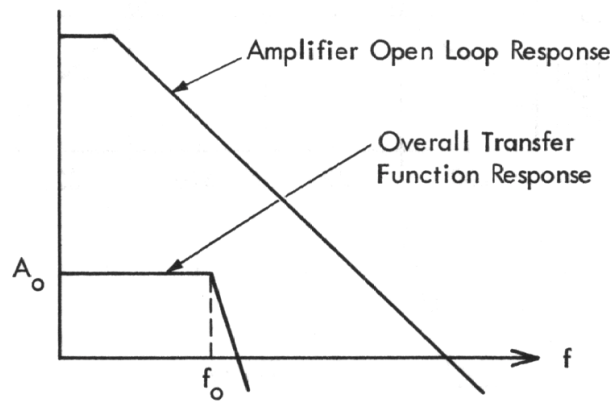
$k = 2.51 \times 10^{-5}$

$C_1 = 0.696 \ \mu F$

$R_1 = R_3 = 15.1 \text{ k}\Omega$

$R_2 = 16.8 \text{ k}\Omega$

CIRCUIT 11: INIC High Pass

$$\frac{E_2}{E_1} = \frac{s^2}{s^2 + \alpha\omega_o s + \omega_o^2} \qquad\qquad \omega_o = 2\pi f_o$$
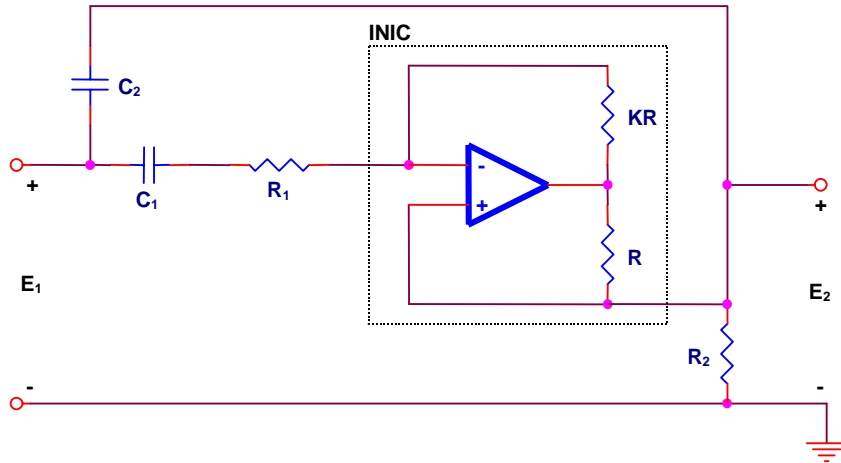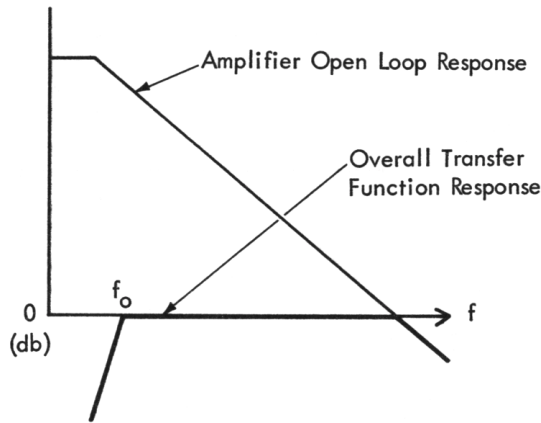
TRANSFER FUNCTION

$$\frac{E_2}{E_1} = \frac{s^2 C_1 C_2 + s(C_2 - KC_1)/R_1}{s^2 C_1 C_2 + s[C_1/R_2 + (C_2 - KC_1)/R_1] + 1/R_1 R_2}$$

Note: Only unity-gain possible with this configuration

Choose : $\alpha \left(= \sqrt{2} \text{ for "max imally flat" 'Butterworth' response}\right)$

Calculate :

$$C_1 = C_2 = \frac{k}{2\pi f_o} \ (\text{defines } k)$$

$$R_1 = \frac{\alpha}{k}$$

$$R_2 = \frac{1}{\alpha k}$$

$$K = 1$$
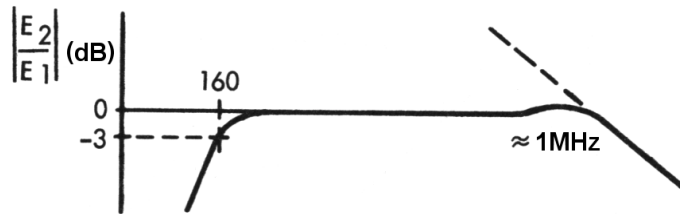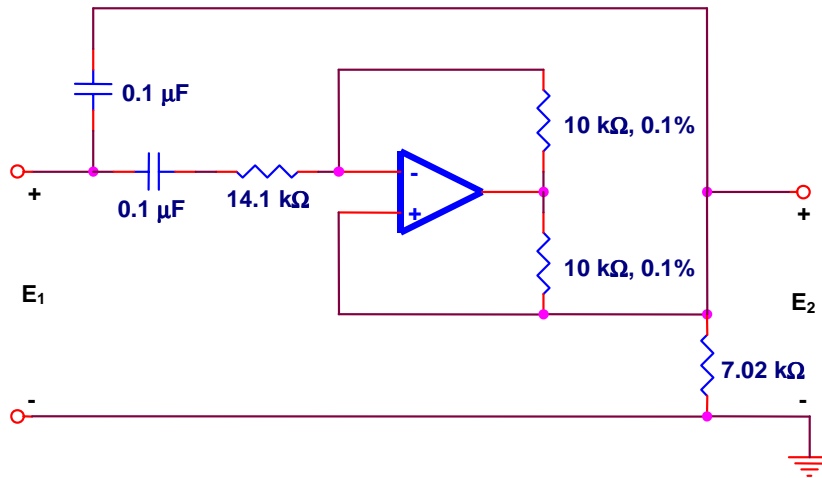
EXAMPLE

Want :

$f_o = 160 \text{ Hz}$

$\alpha = \sqrt{2}$

$C_1 = 0.1 \ \mu F = C_2$

Calculate :

$k = 1.007 \times 10^{-4}$

$R_1 = 14.1 \text{ k}\Omega$

$R_2 = 7.02 \text{ k}\Omega$

CIRCUIT 12: INIC Band Pass

$$\frac{E_2}{E_1} = \frac{-H\omega_0 s}{s^2 + \alpha\omega_o s + \omega_o^{\ 2}}$$

$$\omega_o = 2\pi f_o$$
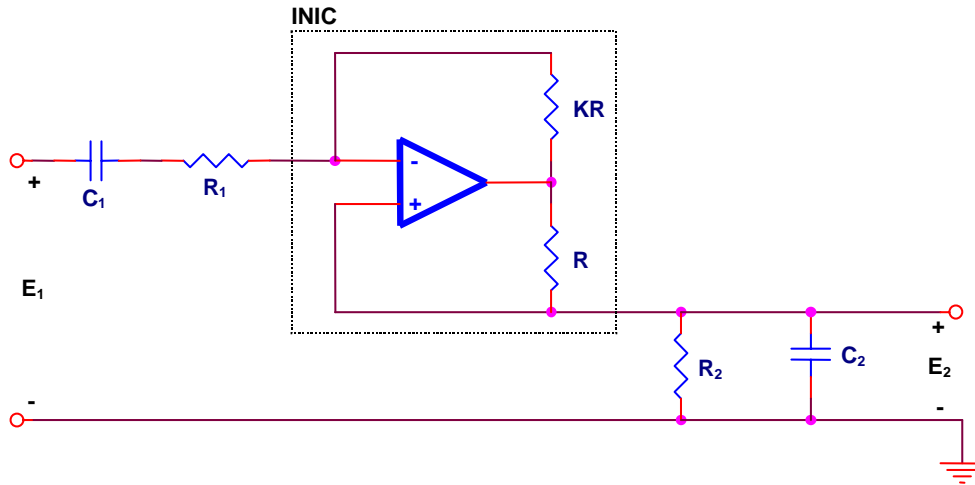
$$A_o = H/\alpha$$

$$Q = 1/\alpha$$
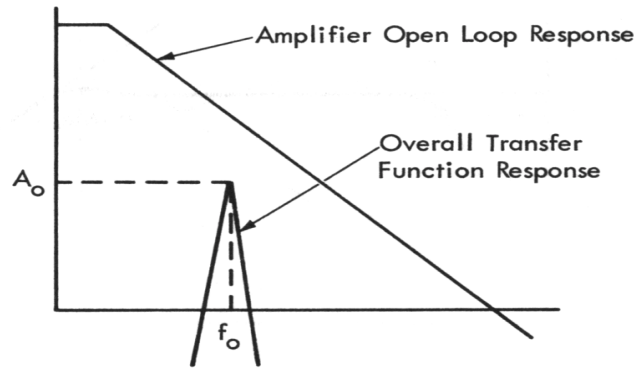
TRANSFER FUNCTION

$$\frac{E_2}{E_1} = \frac{-KsC_1/R_1}{s^2 C_1 C_2 + s(C_2/R_1 + C_1/R_2 - KC_1/R_1) + 1/R_1 R_2}$$

Choose :

$$C_1 = C_2 = \frac{k}{2f_o}\ (\text{defines } k)$$

$$R_1 = R_2 = \frac{1}{k}$$

Note :

$$A_o = \frac{K}{2-K}$$

$$Q = \frac{1}{2-K}$$

$$f_o = \frac{1}{2\pi C_1 R_1}$$

$$K = 2 - 1/Q$$

Note: the choice of R in the INIC is relatively arbitrary, but it should be near 10 kΩ for best results.

INIC

16.67 kΩ

8.33 kΩ

19.9 kΩ   **0.1 μF**

E_1

19.9 kΩ   **0.1 μF** E_2

$Q \approx 100$

200

141

$\left|\dfrac{E_2}{E_1}\right|$

79   80   81
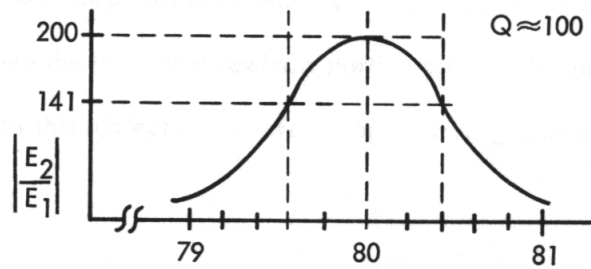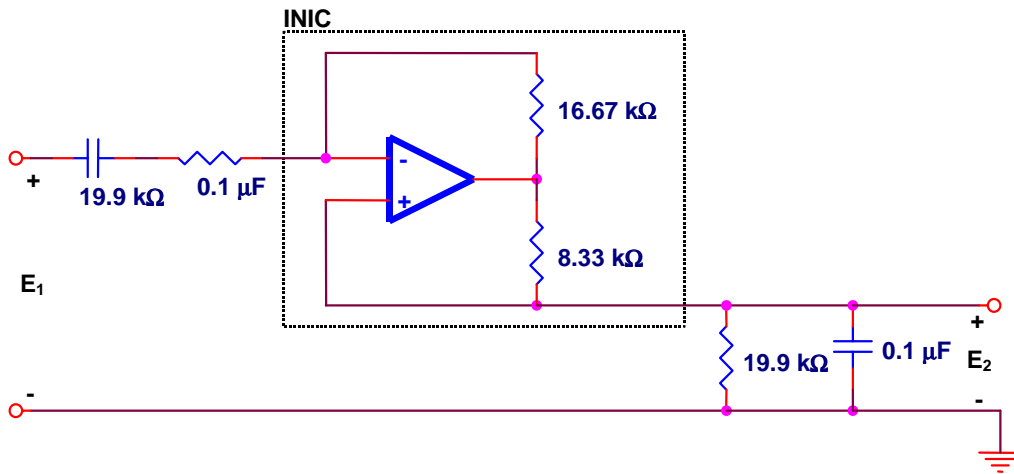
EXAMPLE

Want :

$Q = 100$

$f_o = 80 \text{ Hz}$

$C_1 = 0.1 \ \mu F$

Calculate :

$K = 1.99$

$A_o = 199 \ (46 \text{ dB})$

$C_1 = C_2 = 0.1 \ \mu F$

$k = 50.2 \times 10^{-6}$

$R_1 = R_2 = 19.9 \text{ k}\Omega$

APPENDIX A

References

The literature has many references to the topics in the field of active RC circuits that have been introduced in this handbook. Some of these are given in the list that follows. No attempt has been made to make this an all inclusive list; rather it is presented to give the interested reader a starting place in his pursuit of more de tailed information on this subject. The references are arranged by chapter.

Chapter 1

A general theoretical treatment of the subject of active networks, together with many additional realization techniques may be found in the book by K. L. Su, *Active Network Synthesis*, McGraw-Hill Book Co., Inc., New York, 1965

Chapter 2

General design formulas for several types of bridged-T and twin-T networks, as well as a general discussion of the synthesis of passive RC networks may be found in the book by N. Balabanian, *Network Synthesis*, Chap. 7, Prentice-Hall, Inc., Englewood Cliffs, New Jersey, 1958. A more detailed presentation of several cases, together with several design charts may be found in Appendix VI of the book by C. J. Savant, Jr., *Control System Design*, 2nd Ed., McGraw-Hill Book Co., Inc., 1964

A simplified method for the design of twin-T networks with transmission zeros in the right-half of the complex frequency plane may be found in the article by B. A Shenoi, "A New Technique for Twin-T RC Network Synthesis", *IEEE Transactions on Circuit Theory*, Vol. CT-11, No. 3, pp. 435-436, Sept. 1964

A discussion of the adjustment problem in twin-T networks may be found in the article by K. Posel, "A New Treatment of the RC Parallel-T Network", *Proceedings of the Institution of Electrical Engineer's* (England), Vol. 110, No. 1, pp. 126—138, Jan. 1963.

Chapter 3

A tabular method which may be used to determine the voltage transfer function of a network using a fairly arbitrary feedback network is given in the article by A. G. J. Holt and J. Sewell, "Table for the Voltage Transfer Functions of Single-Amplifier Double-Ladder Feedback Systems," *Electronics Letters* (published by the Institution of Electrical Engineers, England), Vol. 1, No.3, pp. 70-71, May 1965

A design method for a third order filter using a single operational amplifier is given in the article by L. K. Wadhwa, "Simulation of Third-Order Systems with Double-Lead Using One Operational Amplifier," *Proceedings of the IRE*, Vol. 50, No. 6, pp. 1538-1539, June 1962. Articles on similar filters with different zeros may be found in the February and April Proceedings issues of the same year.

Chapter 4

The classic article in this area is the one by R. P. Sallen and E. L. Key, "A Practical Method of Designing RC Active Filters," *IRE Transactions on Circuit Theory*, Vol. CT-2, No. 1, pp. 74-85, March 1955. Some additional circuits are given in the article by N. Balabanian and B. Patel, "Active Realization of Complex Zeros," *IRE Transactions on Circuit Theory,* Vol. CT-10, No.2, pp. 299-300, June 1963

Chapter 5

The properties of a negative-immittance converter as a two-port device were defined by A. I. Larky, "Negative-Impedance Converters," *IRE Transactions on Circuit Theory*, Vol. CT-4, No. 3, pp. 124-131, September 1957. In addition to the circuit for an NIC given in the Larky paper, several other circuit realizations have appeared in the literature. See, for example, D. P. Franklin, "Direct-Coupled Negative-Impedance Converter," *Electronics Letters*, Vol. 1, No. 1, p. 1, March 1965. Power and impedance transformation properties are discussed by L. P. Huelsman, "A Fundamental Classification of Negative-Immittance Converters," *1965 IEEE International Convention Record* Part 7, pp. 113-118, March 1965.

The basic circuit for the realization of a voltage transfer with an INIC presented in this chapter is described in more detail by T. Yanagisawa, 'RC Active Networks Using Current Inversion Type Negative Impedance Converters," *IRE Transactions on Circuit Theory*, Vol. CT-4, No. 3, pp. 140-144, Sept. 1957. Another basic approach that uses VNICs is given by J. G. Linvill, 'RC Active Filters," *Proceedings of the IRE*, Vol. 42, No. 3, pp. 555-564, March 1954.

Chapter 6

Books which cover the topics of this chapter are the ones by Su (see Chapter reference) and L. P. Huelsman, *Circuits, Matrices, and Linear Vector Spaces*, McGraw-Hill Book Co., Inc., New York, 1962.

TEXAS INSTRUMENTS

APPENDIX B

Describing Active Filters

This handbook is addressed primarily to the design of active RC circuits employing operational amplifiers as the active elements. While we have seen that the operational amplifier is extremely versatile in active filter applications, it none-the-less holds true that for every requirement there is one operational amplifier that provides the optimum performance and value. It is, therefore, important far the user to work closely with a reputable manufacturer of operational amplifiers to gain the flexibility of being able to tailor an amplifier to the filter requirement. To fully utilize this service, it becomes necessary for the user to describe his filter requirement.

Describing the Filter

There are many ways to describe a particular active filter. If you have evaluated a circuit from this handbook, referencing the page number along with the amplifier and circuit values employed provides a good starting point for the manufacturer. Any problems or limitations encountered should also be noted.

The shape of the transfer function should be described by function (low pass, band pass, band rejection, high pass), pass-band gain, frequency response (3 dB points or accuracy over range of interest), roll-off characteristics (dB/octave or dB/ decode or rejection at specific frequencies), and type (Butterworth-maximally flat, Bessel-linear phase, Tchebyscheff-equal ripple, etc.). Alternatively, the filter may be described in terms of the basic function of the filter and overall system, a pole-zero plot, or a plot of the desired response versus frequency. Usually some combination of the above method is best used to describe a particular filter.

Optimizing the Circuit

Given a complete description of the active filter requirement, the amplifier manufacturer can begin to make some very interesting design trade-offs. In some band pass and high pass circuits it may be possible to relax the DC voltage and current offset and stability requirements that normally add to the cost of operational amplifiers. In certain filter circuits, the RC phase compensation network inside the operational amplifier may be modified to allow higher frequency performance. Thus, it is often desirable to wed the active amplifier and passive filter elements into a single committed active filter module. The resulting optimized circuit usually provides higher performance at lower overall cost than either a filter employing "standard" amplifiers a "standard" active filter that must be customized at least in terms of cutoff frequencies for your application.

Limiting Specifications

The final filter design can be completely described by electrical, mechanical, and environmental specifications. In the design stage, however, it is important to indicate the critical or limiting specifications so as not to unduly restrict the design add to the design and product cost, and to help resolve any conflicting specifications design trade-offs.

Conclusion

A reputable manufacturer of operational amplifiers with experience and competence in active filters offers valuable services to the user. Whether the service takes the form of circuit consultation, amplifier recommendation, special amplifier design, special filter design, or the supplying of a standard filter, the user must furnish a complete description of his filter requirements.

APPENDIX C

REACTANCE CHART
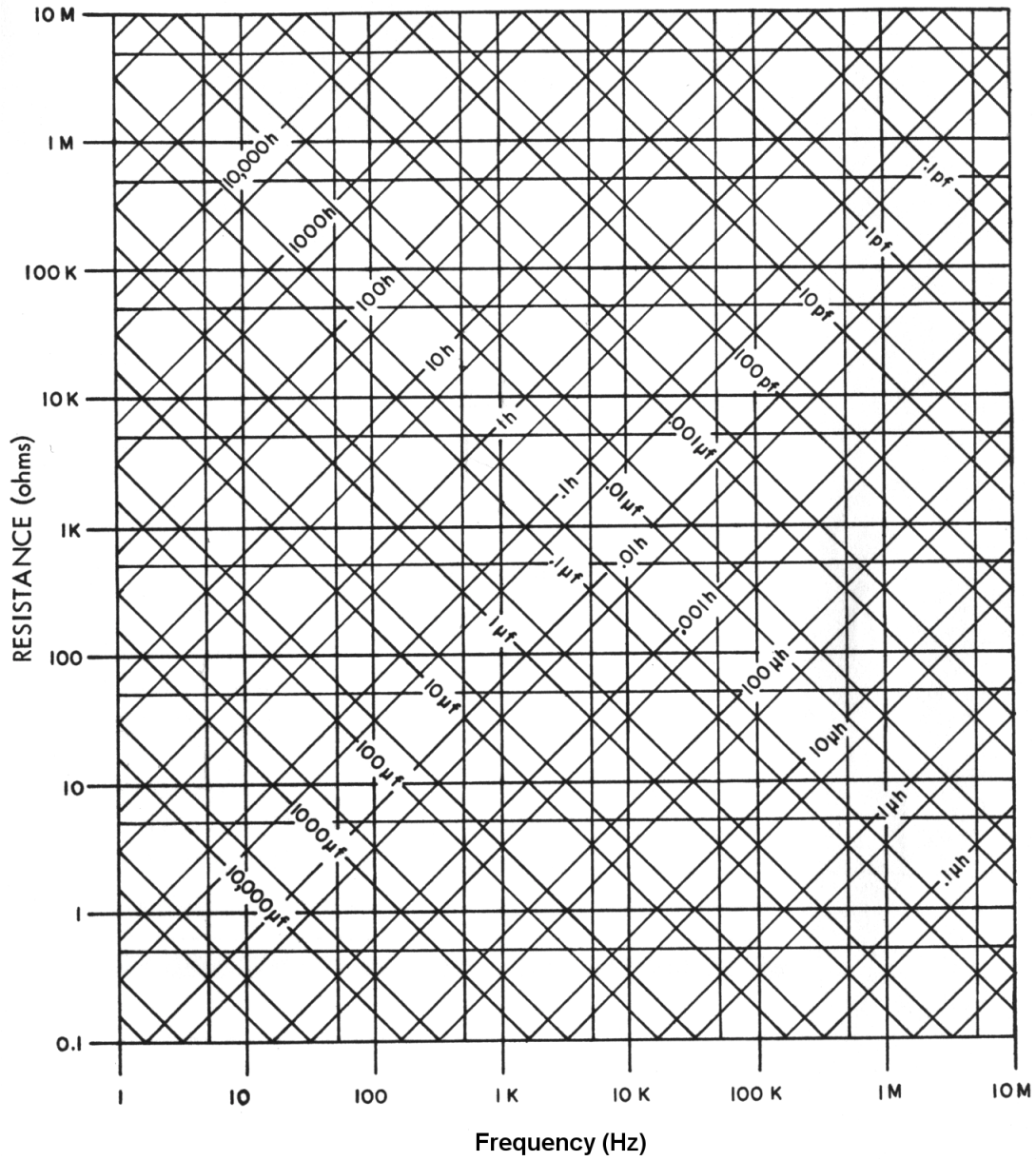
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Texas Instruments
Post Office Box 655303
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